The acoustics of two-dimensional leapfrogging vortex interactions

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The acoustics of low-speed cold shear flows are dominated by the sound produced by vortex interactions. In a two-dimensional jet, these interactions often take the form of coupling between adjacent pairs of counter-rotating vortices. In such a coupling, the spacing of the aft pair tends to narrow, causing it to accelerate, while the forward pair widens and decelerates. The result is a leapfrogging process that continues to repeat indefinitely, or until the pairs merge. In this work, we investigate the acoustics produced by two-dimensional leapfrogging vortex interactions.

Figure 1 illustrates the initial positions and parameters of two vortex pairs. We will present results from simulations for a range of values of the relative vortex strengths, $(\Gamma_2 - \Gamma_1)/(\Gamma_2 + \Gamma_1)$, relative spacing, δ_{y0}/δ_{x0}, and core sizes, r_{10}/δ_{x0} and r_{20}/δ_{x0}. In addition, we will explore the effects of the Mach number of the mean forward propagation velocity, $M_0 = U_0/a_\infty$ and the Reynolds number, $\mathrm{Re} = (\Gamma_1 + \Gamma_2)/2\nu$.

The direct numerical simulation of the compressible near field is performed with the dilating vortex particle method, developed in previous work [1]. Via a Helmholtz decomposition of the velocity field, this method extends the vortex particle method to compressible flows. The computational elements are mass-preserving particles that convect with the local velocity, and carry vorticity, dilatation, enthalpy and entropy strengths that vary in accordance with the Navier-Stokes equations. Derivatives are treated by the method of particle strength exchange, with an eighth-order-accurate kernel in order to faithfully capture acoustic waves of small magntitude. The particle coverage is surrounded by a circular zone of buffer particles that enforce a radiation-type condition to absorb incident acoustic waves. For each case, the initial enthalpy field is established by solution of a Poisson equation, and the initial dilatation and entropy are set to zero.

The acoustic field is derived by a Kirchhoff method, wherein the data on a control surface in the acoustic region of the near-field simulation domain are used to extrapolate the external acoustic field through the integral form of the wave equation. The vortex method simulations are performed in a region that is sufficiently large to include the innermost portion of the acoustic field. The fluctuating stagnation enthalpy and its normal derivative are evaluated on the control surface from the simulation results, and supplied to the Kirchhoff integrals.

As an example of the results we will show, consider the case of two identical pairs initially arranged equidistant from each other ($\Gamma_1 = \Gamma_2$ and $\delta_{y0}/\delta_{x0} = 1$). The initial vorticity of each vortex is Gaussian-distributed, with $r_{10}/\delta_{x0} = r_{20}/\delta_{x0} = 0.15$. The Reynolds number is 10^4, and the Mach number of the mean forward motion is 0.108. The panels of Figure 2 depict the vorticity and resultant acoustic field of a single leapfrogging event, observed in a reference frame that moves with the system. As the aft vortex pair is threaded through the forward pair, a large four-lobed pressure pulse is generated, immediately followed by a second one of opposite sign. Each of these subsequently radiates outward. The outer portion of the pressure field in each panel contains the acoustic pulse produced by the previous event. Note that the region between pulses is relatively silent. Also, the forward-propagating pulses encompass a larger sector than the rearward-propagating ones, due to a slight fore-to-aft convection induced by the dipolar vortex system.

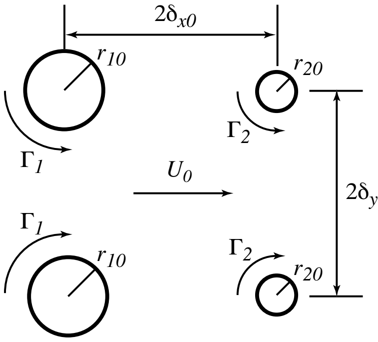

Figure 1. Schematic of initial vortex positions

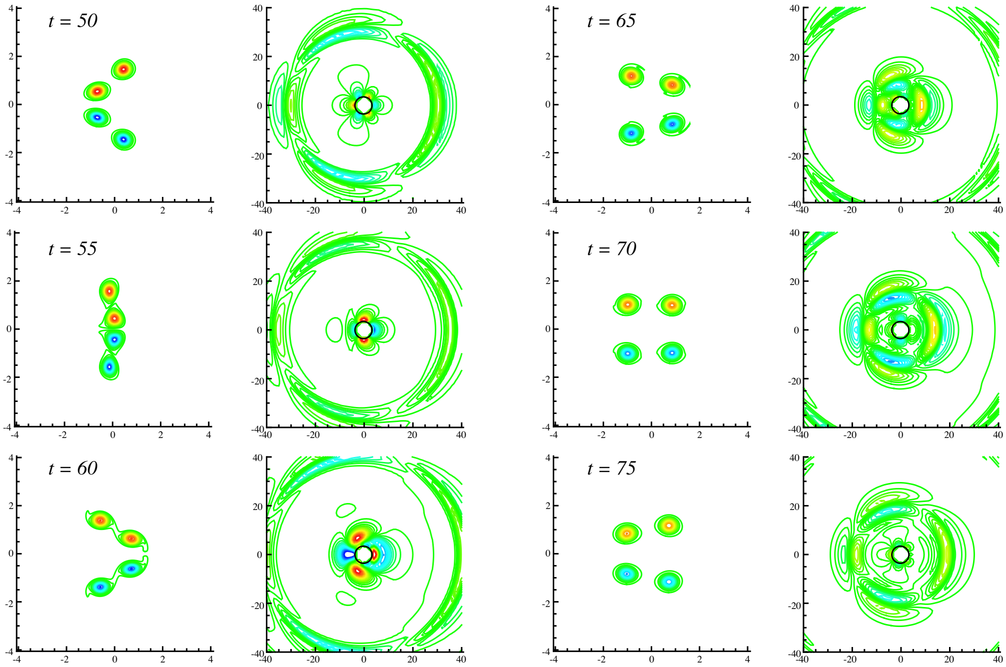

Figure 2. Vorticity (left) and pressure (right) at six instants. Kirchhoff surface shown as black circle at center. Time is scaled by a_∞ and δ_{x0}.

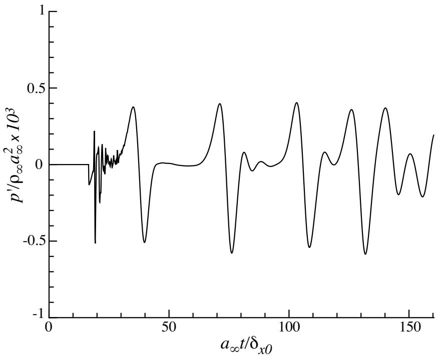

Figure 3. Pressure history at $(x/\delta_{x0}, y/\delta_{x0}) = (0, 20)$.

The pressure history from this process, observed at a single point at $(x/\delta_{x0}, y/\delta_{x0}) = (0, 20)$, is shown in Figure 3. After the initial time lag, the usual nonphysical transient—a consequence of the near-field dilatation adjusting to the correct level—persists for approximately 15 time units. This is followed by a narrow pulse of positive and negative pressure, resulting from the first leapfrogging event; the second event produces a pulse observed at $a_\infty t/\delta_{x0} = 70$. Also apparent are some small, higher frequency oscillations at $a_\infty t/\delta_{x0} = 80$, due to the nutation of the non-circular vortex cores. As time progresses, the quiet interval between the pulses becomes shorter. As the vortex pairs begin to couple, they move closer together and their leapfrogging accelerates. At $a_\infty t/\delta_{x0} = 140$, the cycle is broken by a large positive pressure followed by smaller oscillations, as the vortex pairs begin to merge. The resulting single counter-rotating pair is a much less efficient emitter.

We will present results such as these from simulations for a wide range of the parameters. Particular attention will be devoted to the kinematics of the merging process, and the acoustics generated by it. We will show that the acoustics of this process are similar to those produced by a pair of counter-rotating elliptical vortices with shrinking core aspect ratio.

References

[1] J. D. Eldredge, T. Colonius, and A. Leonard. A vortex particle method for two-dimensional compressible flow. *J. Comput. Phys.*, 179:371–399, 2002.

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