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Why Parameterizing Element Connectivity for Topology Optimization?

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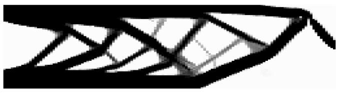
Summary Recently, a new topology optimization formulation, called the element connectivity parameterization (ECP), has been proposed. The objective of this work is to present the motivation for the development of ECP, the idea behind it and its applications to some topology optimization problems which are otherwise difficult to solve. We begin with the issue of unstable elements for topology design problems involving nonlinear analysis and ascribe the source of the numerical problem to the intermediate density element of the conventional topology optimization formulation. To overcome the numerical problem, the degree of element connectivity is used to describe structural layouts while finite elements remain solid during whole topology optimization iterations. A few numerical problems are considered to show the usefulness of ECP.

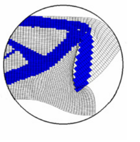
1. Element Instability in Nonlinear Problems

Topology optimization formulation using the element density as the design variable has been very successful in a wide class of design problems [1]. However, it suffers from a numerical instability problem resulting from the so-called unstable element in topology optimization involving nonlinear analysis. The application of the element density-based formulation may yield physically incorrect design as indicated in Fig. 1. Fig. 1 shows a typical result for an end-compliance minimization problem considering geometrically nonlinear deformations.

The result in Fig. 1 may be obtained by the direct application of the convergence relaxation method [2]. As shown in Fig. 1, the low-density elements, especially near the loaded end, experience excessively large deformations causing numerically negative element areas, so they become unstable elements. The appearance of the unstable elements causes solution oscillations and poor convergence in the Newton-Raphson iteration. The low-density related problem has motivated the development of a new stable method.



(a) Optimal Layout

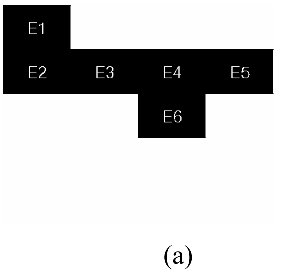


(b) Deformed Shape

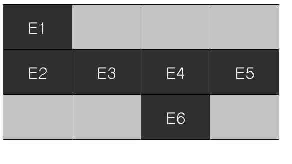
Fig. 1 An optimized result by a direct application of the element-density based topology optimization formulation for an end-compliance topology optimization considering geometrically nonlinear deformations.

2. Element Connectivity Parameterization

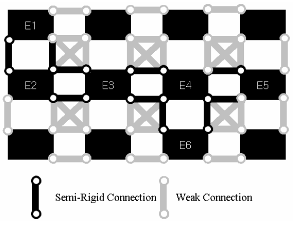
To resolve the problem discussed above, a so-called element connectivity parameterization method [3] has been propose. The best way to describe is, perhaps, to compare how a structural layout in Fig.2 (a) is modeled by the standard element-density method and the element connectivity parameterization (ECP) method. To define the structural topology in Fig. 2(a), the element-density method uses low-density elements while the ECP method introduces semi-rigid zero-length links connecting elements. As in the SIMP method, the stiffness of the link in the ECP method is treated as the function of the link density and is appropriately penalized. Since the element connectivity is used to define structural layouts, all finite elements discretizing the design domain can remain solid during whole design optimization iterations and the appearance of the unstable elements can be prohibited. Fig. 3 illustrates the element connection by a link *i* and the penalization of its stiffness k_i^{pq} connecting local nodes *p* and *q*.



(a)

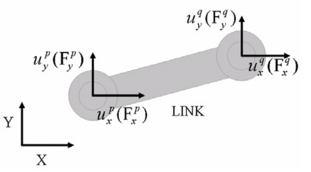


(b)



(c)

Fig. 2 Structural layout modeling. (a) A typical structural layout, (b) modeling by the standard element density method (Grey: Weak elastic elements, Black : Solid elements), (c) the modeling by the ECP method (Black Line : Semi-Rigid Connection: Grey Line : Weak Connection).



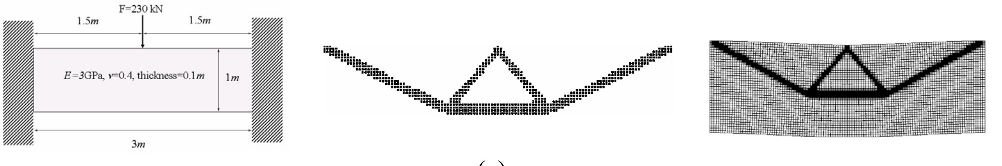
$$k_i^{link} = l_0(\gamma_i^{pq})^n k_{link}$$

$$\gamma_{i,LOW}^{pq} \le \gamma_i^{pq} \le \gamma_{i,UPPER}^{pq} = 1$$

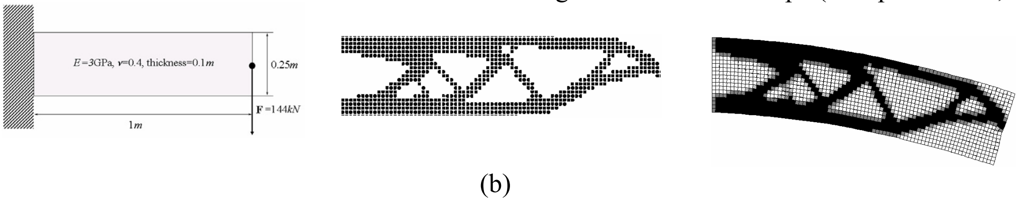
Fig. 3 Link element connecting nodes *p* and *q* at connection *i*. It has the design variable γ_i^{pq} .

3. Numerical Examples

To illustrate the effectiveness of the ECP method, two topology design optimization problems described in Fig. 4 and Fig. 5 are considered. Fig. 4 shows a well-known compliance minimization problem while Fig. 5 shows the topology design problem of a micro electro-thermal-compliant actuator. In both problems, geometrically nonlinear deformations are considered. Though not sated in details here, numerical problems such as solution oscillations, convergence difficulty due to the unstable elements were not observed during the optimization iterations with the ECP method. The skeleton images in the figures denote the images representing the distribution of the density of the artificial link; the density varies from 0 to 1 where the value of 1 corresponds to the rigid connection. In plotting the deformed shapes, the value of the link density has been properly translated as the equivalent value of the finite element density; the translation is done after the design optimization is completed.



(a)
Skeleton image Deformed shape (Compliance=12,549J)



(b)
Skeleton image Deformed shape (Compliance=26,510J)

Fig. 4 A compliance minimization of geometrically nonlinear structures. (a) Mass constraint ratio=10%, 120×40 4-node bilinear finite elements used, (b) mass constraint ratio=50%, 80×20 4-node bilinear finite elements used.

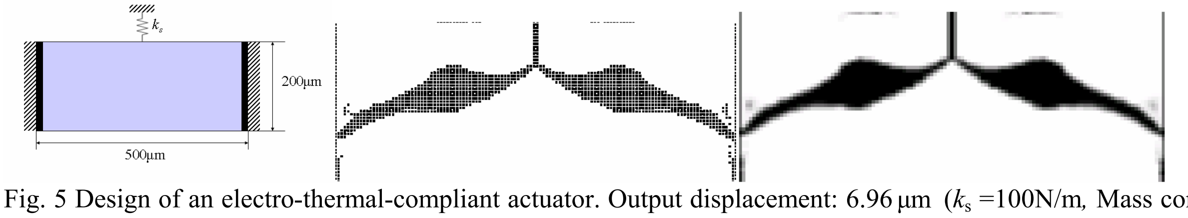


Fig. 5 Design of an electro-thermal-compliant actuator. Output displacement: 6.96 μm (k_s =100N/m, Mass constraint = 30%, Young's Modulus = 200 GPa, Poisson ratio = 0.31, thickness=15 μm , the electric conductivity= $6.4 \times 10^6 K(\Omega m)^{-1}$, the thermal conductivity=90.7 W/(Km), the convection coefficient=18.7×10^3W/(m^2K), thermal expansion coefficient=$15 \times 10^{-6}K^{-1}$, applied voltage = 0.3 volt).

4. Conclusions

The element connectivity parameterization method was investigated with specific applications in a nonlinear compliance minimization problem and a nonlinear electro-thermal-compliant actuator design problem. By defining a structural layout inside a design domain through the degree of the element connectivity, low-density unstable finite elements were avoided. In this approach, the discretizing finite element remained solid during the whole design iterations. The element connectivity was parameterized by artificial zero-length one-dimensional links and their stiffnesses were penalized for clear layouts. Work is in progress to extend ECP for stress-constrained design problems.

5. REFERENCES

[1] Bendsøe, M.P., and Sigmund, Optimization of Structural Topology, Shape, and Material, Springer-Verlag, New York. 2003.

[2] Buhl T, Petersen CBW, Sigmund O. Stiffness design of geometrically nonlinear structures using topology optimization. *Structural and Multidisciplinary Optimization* 2000; **19**(2):93-104.

[3] Yoon Gil Ho, Kim Yoon Young. Inter-element connectivity parameterization for topology optimization of geometrically nonlinear structures. Submitted, 2003.