WAVES OF DEFORMATION PROPAGATION IN NONLINEAR VISCOUSLY ELASTIC POROUS MATERIAL

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Summary A mathematical model of an isotropic viscously elastic porous medium is studied in this paper. Cavities are spherical and are quasi-equally spaced in the medium. Distance between cavities is a lot larger than the average radius of cavities and in turn is a lot smaller than the length of the wave that propagates in the medium. Dependencies of velocity and damping of the wave on the parameters of porosity of the material have been established and the analysis carried out. The phenomenon of non-linear stationary wave of deformation propagation has been studied for the material also and the dependencies of it's parameters on porosity have been established.

PRELIMINARY ASSUMPTIONS

A mathematical model of dynamics of porous medium has been proposed in [1]. The following assumptions were made. Studied medium is firm isotropic and viscously elastic; pores are spherical and equally spaced. Propagating wave has finite amplitude, which means that geometrical, physical and porous nonlinearities are taken into account. Distance between cavities L is a lot larger than radius of a cavity R_0 $(L >> R_0)$ and in turn is a lot smaller than the length of a wave Λ $(L << \Lambda)$, so there can be no interaction between the cavities. We consider, that the propagating wave is quasi-longitudinal, so we can state that pressure on a cavity is caused by longitudinal stress $\sigma_{33} = (\lambda + 2\mu)\frac{\partial u_3}{\partial x_3} - (\lambda + 2\mu)z$. In this expression $z = NV$, where N is a number of cavities in the volume, V is a volume of a cavity, besides that $V = V_0 + V'$, where V_0 is a starting volume of a cavity, V' is a volume of a cavity indignant by a wave, λ and μ are Lame coefficients, considering $\mu < \lambda$. Pressure in cavities is neglected.

DISPERSIVE PROPERTIES

Longitudinal wave propagation in porous medium lengthways the x_3 axis can be described using the following combined nonlinear equations (since one-dimensional problem is studied in this paper, the x_3 coordinate can be denoted by x, and longitudinal constituent of the displacement vector u_3 is denoted by u):

$$\begin{cases} \rho_0 \dfrac{\partial^2 u}{\partial t^2} = (\lambda + 2\mu)\dfrac{\partial^2 u}{\partial x^2} - N(\lambda + 2\mu)\dfrac{\partial v}{\partial x} + b\dfrac{\partial^3 u}{\partial t\, \partial x^2} + \dfrac{P}{2}\dfrac{\partial}{\partial x}\left(\dfrac{\partial u}{\partial x}\right)^2 \\ \ddot{v} + \omega_0^2 v - \dfrac{R_0}{c_l}\dddot{v} - Gv^2 - \beta_1\left(2v\ddot{v} + \dot{v}^2\right) = (\lambda + 2\mu)\dfrac{4\pi R_0}{\rho_0}\left(\dfrac{\partial u}{\partial x} - Nv\right) \end{cases} \tag{1}$$

The first equation describes propagation of a plane longitudinal wave in medium with pores taking into account that each pore volume is changing. The second one describes oscillatory process of cavity volume changing caused by the material deformation.

In these equations ρ_0 is a starting density of the material, $\omega_0^2 = 4\mu / \rho_0 R_0^2$ is a square of resonant frequency of cavity volume oscillations and $c_l^2 = (\lambda + 2\mu)/\rho_0$ is a square of longitudinal velocity. The following are some additional notations: $G = 11\omega_0^2 / 16\pi R_0^3$, $\beta_1 = 1/8\pi R_0^3$ and $P = (4\mu + 3\lambda + 2A + 6B + 2C)$, where P is a coefficient, caused by geometrical and physical nonlinearities, A, B, C are Landau constants of third order.

From these joint equations (1) it can be seen, that the consequences of the existence of cavities are the dispersion of a wave (frequency-dependent wave propagation velocity) and an additional nonlinear effect (so-called cavitary nonlinearity). These factors manifest in different regimes.

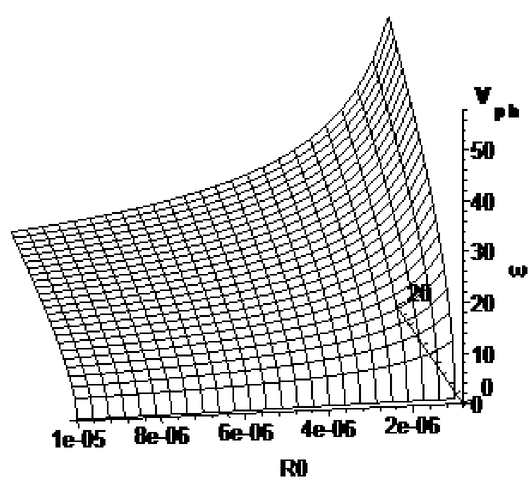

Fig. 1. Phase velocity dependence on porosity

Expressions for phase velocity and coefficient of damping for the propagating wave were obtained and are given graphically on figures 1 and 2 respectively.

During the analysis we came to the conclusion, that taking both viscosity of the material and dissipation of the wave due to porosity into account causes an additional dispersive component to appear. Also analysis was carried out separately for the case of viscosity present with no dissipation from the cavities and vice versa to compare their contributions.

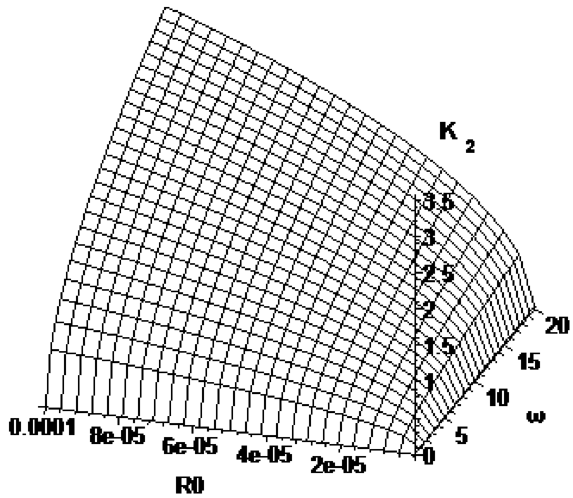

Fig. 2. Coefficient of damping dependence on porosity

SOLITON OF DEFORMATION

From (1) it follows that the propagating wave, in terms of this model, is influenced by both dispersion and nonlinearity. Non-linearity causes generation of additional harmonics to which the energy is being swapped, which mainly causes drops in the profile of the wave, while dispersion causes smoothing of the drops because of the difference in wave velocities. The combined effect of these two factors can be a reason for forming nonlinear stationary waves. These are the waves that propagate with constant velocity without changing its form.

We seek for a solution of (1) in the form of stationary wave of deformation $W(\xi = x - Vt) = U$, propagation of which is described by the following equation $\frac{\partial^2 W}{\partial \xi^2} + aW + bW^2 = 0$, where V is the velocity of the stationary wave and ξ is a new coordinate.

Physically realizable are only the cases when the wave form does not have any constant constituent. In this particular case it is possible only if wave velocity changes in the ranges $\sqrt{\frac{\omega_0^2}{4\pi R_0 c_l^2 N + \omega_0^2}} < V < 1$, which is that $a < 0, b > 0$. Expressions for amplitude of the soliton and width of the soliton were obtained and analyzed. Figures 3 and 4 respectively represent amplitude and width dependence on the porosity.

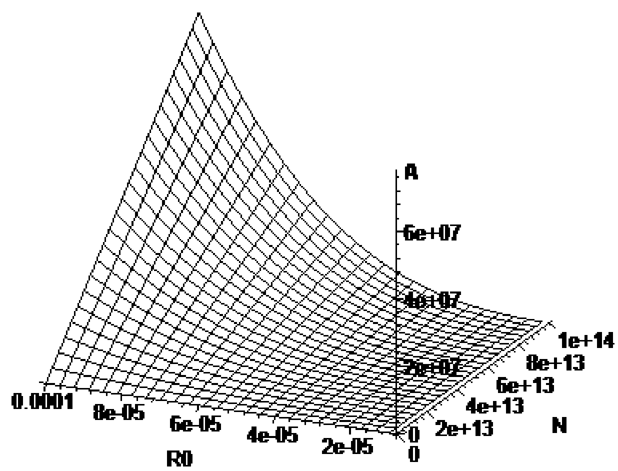

Fig. 3. Soliton amplitude dependence on porosity

CONCLUSIONS

Dependencies of velocity of the wave on the parameters of porosity of viscously elastic material have been established and the analysis carried out.

The phenomenon of non-linear stationary wave of deformation propagation has been studied for the material also and the dependencies of basic parameters of the soliton on porosity have been established.

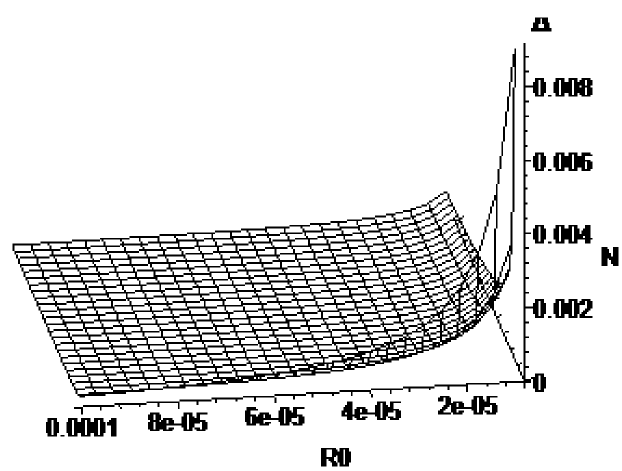

Fig. 4. Soliton width dependence on porosity

References

[1] Bagdoev, A. G. and Shekhoyan, A. V., *Nonlinear waves in the solid viscous medium with pores*, Acoust. Physics, 2 (1999) 149-156.