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A SYSTEMATIC MODEL REDUCTION METHOD FOR THE CONTROL OF FLEXIBLE MULTIBODY SYSTEMS

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Summary This paper presents a method to build closed-form dynamic equations for flexible multibody systems with a minimal kinematic description. Relying on the Finite Element formalism, the method is able to tackle complex topologies with closed-loops in a systematic way. Thus, it will be of great use in the framework of model based control of flexible mechanisms. The reduction procedure is based on an interpolation strategy: a Finite Element model is built and reduced for a number of selected points in the configuration space, and then, a piecewise polynomial model is adjusted to match the collected data. A few applications of the reduction method are considered: a serial pick-and-place machine, a flexible four bar mechanism, and a parallel kinematic manipulator.

STATE OF THE ART

The development of parallel kinematic mechanisms is a major advance in the field of high-speed machine-tools and manipulators: the motorized joints can be located on the base so that the moving mass is reduced, and faster motions are possible. However, the extended bandwidth of the actuators may excite the natural vibration modes of the system so that the deformations have a significant influence on the performances.

Flexible mechanisms with parallel architecture exhibit a complex kinematic and dynamic behaviour which requires powerful control algorithms, usually based on the input-output feedback linearization of the dynamic equations. This technique is performed with an inverse dynamic model, which outputs the required joint actions given a set of joint values, rates and accelerations. In fact, this algebraic relationship is directly connected to the Ordinary Differential Equations (ODEs) of the underlying state-space model. Thus, the design of the control algorithm requires a high-quality mechanical model in terms of accuracy, efficiency, and conciseness.

Modeling methods in flexible multibody dynamics may be grouped into two classes. On the one hand, assumed modes based methods, such as the recursive Lagrangian formulation presented by Book [1] and the symbolic non-recursive method introduced by Cetinkunt and Book [3], are able to provide compact closed-form descriptions of open kinematic chains, which are very useful for control. But their extension to more complex topologies is less attractive, since closed-loops introduce algebraic constraints between redundant variables. The system is then described by Differential Algebraic Equations (DAEs), and a numerical constraint elimination technique is necessary to construct an underlying set of ODEs. On the other hand, numerical Finite Element (FE) methods [5] are systematic and general in nature, and they are able to handle complex topologies and highly flexible components. However, the generated DAEs contain many redundant variables and the constraint elimination involves expensive computations. Thus, none of these methods meets the requirements for model based control of flexible mechanisms with complex parallel architecture.

SUMMARY OF THE REDUCTION TECHNIQUE

In this work, a systematic and general methodology has been developed to provide closed-form ODEs for such flexible mechanisms with complex topology. The method relies on two ingredients: a numerical reduction procedure and an interpolation strategy. The authors have already demonstrated the relevance of this philosophy for rigid mechanisms [2]. This paper proposes a generalization to flexible multibody systems.

Starting from a FE model, a component mode synthesis is applied to build the reduced set of ODEs in any given configuration. Then, an explicit description of the reduced model is obtained by interpolation in the whole configuration space. Selecting the number of component modes and the interpolation precision level, the user is able to balance accuracy against efficiency and conciseness. The procedure, illustrated in figure 1, is performed offline, so that the closed-form dynamic equations are almost directly available online for the control algorithm.

Numerical reduction

In structural dynamics, the well-established Craig-Bampton and Mc-Neal/Rubin component mode methods [4, 7] allow to reduce the order of a linear model. They have been extended in the framework of flexible multibody dynamics (see [5]), so that a substructure with a linear elastic behaviour within a local frame can be represented by a low-order modal model.

In this research, we exploit those substructuring techniques to build the linearized reduced-order model of a full mechanism in a given configuration. As we are concerned by control, the actuated and sensed degrees of freedom represent a natural choice for the boundary degrees of freedom that should be kept in the reduced model. In general, the remaining internal vibration modes associated with this local model depend on the configuration. This means that the system is described by a time-varying modal basis, and that the physical interpretation of the modal amplitudes also changes during

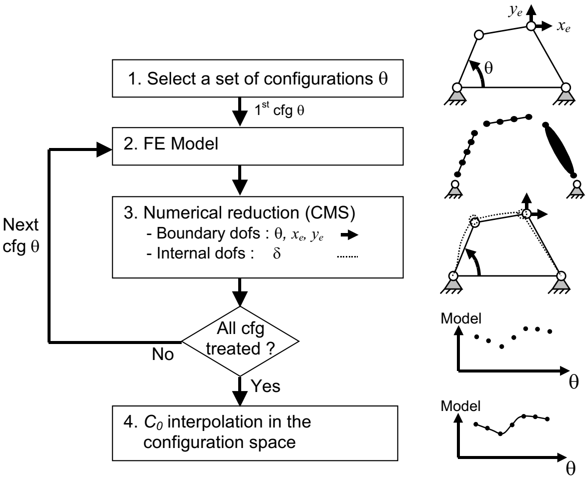

Figure 1. Basic principle of the reduction method. Illustration: a flexible four bar system, where the effector motion (x_e, y_e) is controlled at the joint angle θ. The configuration space is 1-dimensional.

the motion. To guarantee the continuity of the model, a modal tracking strategy is elaborated on the basis of the Modal Assurance Criterion (MAC value) defined in structural identification.

Interpolation

All the coefficients of the linearized reduced model may be collected in a vector f, which is a function of the configuration variables θ. As in response surface optimization methods [6], the construction of an explicit model $f(\theta)$ from sampled data ($f_i(\theta_i),\ i = 1...r$) can be achieved with an efficient interpolation strategy in the n-dimensional configuration space.
Among interpolation strategies, we can distinguish the global ones, based on a high-order interpolation function (neural networks, rational approximations, kriging...) and the local ones, based on a collection of simple interpolation functions defined on a tesselation of the configuration space.
The computational efficiency being a major issue for real-time control, the second family is retained in this research. In general, those local piecewise methods suffer from discontinuous transitions. But some interpolation functions, such as the Lagrange polynoms, yield a global C_0 description. In this paper, several polynomial functions are compared from the point of view of accuracy, continuity, computational load and memory size required to store the model.

CONCLUSIONS

A model reduction method has been developed for flexible multibody systems, which allows to build suitable models for control from Finite Element models. The user is able to adjust the trade-off between accuracy, efficiency and conciseness. The relevance of the method is demonstrated with three examples: a serial pick-and-place machine, a flexible four bar mechanism, and a parallel kinematic manipulator.

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