

GENERATION OF INTERNAL WAVES IN THE DEEP OCEAN BY BAROTROPIC TIDES

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INTRODUCTION

By itself, the heating and cooling of the ocean surface would only be capable of driving a significant flow in a thin surface layer ("Sandström's theorem"). The flow below this layer must be mechanically driven, and the two possible sources of mechanical energy are the winds and the tides [1]. In the main thermocline, the winds play a dominating role, mainly via Ekman pumping, but in the abyss below, the tides are probably more important.

When tidal currents encounter rough topography on the bottom of the ocean, internal waves are created, and when these waves steepen and break through nonlinear interactions, they cause vertical mixing. This mixing is what drives the deep circulation. It is therefore of great interest to calculate the energy flux from the tides to the internal wave field.

THEORY

A general expression for the total energy flux C to the internal waves has been derived by Bell [2] and Llewellyn Smith and Young [3]:

$$C = C_+ + C_-, \quad (1)$$

where

$$C_+ = \frac{\rho_0 N U_+^2}{8\pi^2} \sqrt{1 - \frac{f^2}{\omega^2}} \iint \frac{k_x^2}{k} |\tilde{h}(\mathbf{k})|^2 d\mathbf{k} \quad (2)$$

and C_- is given by a similar expression with U_+ replaced by U_- and k_x by k_y . Here ρ_0 denotes the mean density of sea water, N the buoyancy frequency, f the Coriolis parameter, ω the angular frequency of the tide, and $\mathbf{k} = \hat{x}k_x + \hat{y}k_y$ and $k = |\mathbf{k}|$. The velocities U_+ and U_- are the major and minor axes of the tidal ellipse, respectively, and the x -axis has been chosen to lie along the major axis. $\tilde{h}(\mathbf{k})$ denotes the Fourier transform of the bottom topography:

$$\tilde{h}(\mathbf{k}) = \iint e^{-i\mathbf{k}\cdot\mathbf{r}} h(\mathbf{r}) d\mathbf{r},$$

where $\mathbf{r} = \hat{x}x + \hat{y}y$.

In eq. (2) the total energy flux has been written as an integral over spectral space. Equivalently, it may be written as an integral over real space. The product of the two functions k^{-1} and \tilde{h} in spectral space then translates to a convolution integral between the two corresponding inverse transforms, $(2\pi r)^{-1}$ and h , respectively. We obtain

$$C_+ = \iint P_+ d\mathbf{r}, \quad (3)$$

where the energy flux density is given by

$$P_+(\mathbf{r}) = \frac{\rho_0 N U_+^2}{4\pi} \sqrt{1 - \frac{f^2}{\omega^2}} \frac{\partial h}{\partial x} \iint \frac{1}{|\mathbf{r} - \mathbf{r}'|} \frac{\partial h(\mathbf{r}')}{\partial x'} d\mathbf{r}'. \quad (4)$$

A number of simplifying assumptions were made in the derivation of eq. (2). The flow was assumed to be hydrostatic, the buoyancy frequency uniform, the tidal excursion much smaller than the horizontal scale of the topography, and the bottom slope subcritical (which is necessary for the linear wave theory to be valid). Finally, the ocean was assumed to be infinitely deep. The assumptions of vertically uniform N and infinite depth were removed in a recent study by Llewellyn Smith and Young [3]. In an ocean of finite depth, the internal waves can be expressed as a discrete series of vertical modes. The most essential effect of this is that the wave generation by topographic scales longer than the horizontal wavelength of the first mode is suppressed. The criterion for this to occur can be written $L > ND/(\omega^2 - f^2)^{1/2}$, where L is the topographic scale and D the ocean depth. For vertically nonuniform N , WKB-analysis shows that N in this expression should be replaced by its vertical average \bar{N} , i.e. we replace ND by $\int_0^D N(z) dz$.

In numerical calculations, it is impractical to use the exact expression, involving an infinite series. We therefore take the finite depth into account in an approximate way, by multiplying the integrand in eq. (2) by a spectral high-pass filter $\alpha(ak)$, with the properties $\alpha(\kappa) \approx 1$ for $\kappa \gg 1$ and $\alpha(\kappa) \approx 0$ for $\kappa \ll 1$. The cutoff length a is proportional to the wavelength of the first mode. Both a rational filter $\alpha(\kappa) = \kappa^2/(1 + \kappa^2)$ and a Gaussian filter $\alpha(\kappa) = 1 - \exp(-\kappa^2)$ have been tried, with very little difference in the results.

After inverse Fourier transformation, the effect of the high-pass filter is that the function $1/r$ in the integrand of eq. (4) is replaced by the convolution of $1/r$ and the inverse Fourier transform of the filter function α . Denoting the result of this

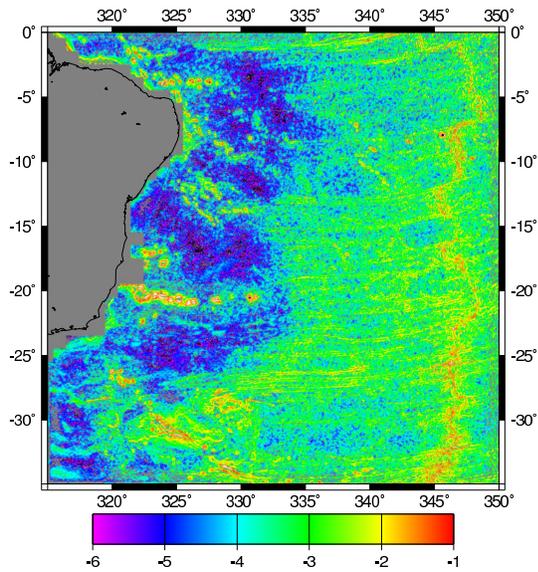


Figure 1. Logarithm of the energy flux density in W/m^2 from the M2 tide to internal waves in the Brazil Basin (e.g. -3 means 10^{-3}W/m^2).

convolution $g_a(r)$, the resulting final expression for the flux density is

$$P_+(\mathbf{r}) = \frac{\rho_0 N_B U_+^2}{4\pi} \sqrt{1 - \frac{f^2}{\omega^2} \frac{\partial h}{\partial x}} \iint g_a(|\mathbf{r} - \mathbf{r}'|) \frac{\partial h(\mathbf{r}')}{\partial x'} d\mathbf{r}', \quad (5)$$

where N_B is the buoyancy frequency at the bottom. The function $g_a(r)$ is approximately equal to $1/r$ for $r \ll a$, while $g_a(r) \sim r^{-3}$ for $r \gg a$. For the rational and Gaussian filter functions used here, g_a can be expressed analytically in terms of integrals of Bessel functions. The cutoff length a is defined by

$$a = \frac{\beta \bar{N} D}{\pi \sqrt{\omega^2 - f^2}},$$

where β is a numerical coefficient of order unity that is determined by comparison with an analytically solvable test problem with a simple ridge topography. The contribution P_- from the minor tidal component is given by a similar expression as eq. (5), with U_+ , x and x' replaced by U_- , y and y' , respectively.

RESULTS

The energy flux density $P = P_+ + P_-$ is calculated numerically from eq. (5) and the corresponding equation for P_- . In order to evaluate P in one point, one must integrate over a region around this point. Since $g_a(r)$ decreases rapidly beyond the cutoff length a , the integral was truncated at a distance $5a$ from the evaluation point. For the real ocean, a is typically a few tens of kilometers or less. Over this length scale, N , U_\pm and f can be considered horizontally uniform, as assumed in the derivation of eq. (2). (This issue would be more problematic if a calculation in spectral space were attempted.)

The input data used in the calculations were the Smith and Sandwell bottom topography with a nominal resolution of $1/30$ degrees, the Egbert tidal velocities, and the buoyancy frequency obtained from the SAC hydrographic database. Calculations for the global ocean are underway, but have not yet been completed. Results for the Brazil Basin are shown in Fig. 1. The calculated total energy flux for the M2 tide over the area shown is approximately 26 GW. The main uncertainty is due to unresolved topography. It can be seen from other calculations with an additional low-pass filter that most of the energy flux is generated by topographic scales below 10 km, and it therefore seems very likely that a larger energy flux would be obtained with a higher topographic resolution. Another uncertainty is the effect of supercritical slope. For the area shown in the figure, the total energy flux from points where the slope is supercritical (and the theory therefore invalid) is approximately 12 GW.

References

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