XXI ICTAM, 15–21 August 2004, Warsaw, Poland

LARGE EDDY SIMULATION OF RAYLEIGH-BÉNARD CONVECTION IN AN INFINITE FLUID LAYER

A. Sergent*, P. Joubert**, P. Le Quéré*
*LIMSI, CNRS, BP 133, 91403 Orsay, France
**LEPTAB, Université de La Rochelle, Avenue M. Crépeau, 17042 La Rochelle cedex 1, France

Summary A numerical study of Rayleigh-Benard convection in an infinite fluid layer (Pr=0.71) is performed using large eddy simulation (LES) of the Navier-Stokes equations with the Boussinesq approximation. We present results in the 'hard turbulence' regime ($2.10^5 < Ra < 2.10^9$). The LES modelling uses the mixed scale diffusivity model, that we have originally developed in the case of the differentially heated cavity. The main observation is the ability of the computations to reproduce the 2/7 scaling behavior over a large Ra range ($2.10^5 < Ra < 2.10^8$). Moreover the regime transition towards the 'ultra-hard regime' is observed at $Ra = 2.10^9$.

INTRODUCTION

Much controversy surrounds the physics of turbulent Rayleigh-Bénard convection, especially at high Rayleigh numbers with recent experiments [1] [2] [3] or computations [4] [5]. The focus is on the Nusselt number correlations ($Nu \propto Ra^n$) and the successive transitions towards a steeper power laws.

As a transition of the mean flow is considered to be probably responsible of the turbulent regime transition between the 2/7 and steeper power law regime[1][4], numerical studies can be seen as an interesting complement to experiments in order to provide a physical picture of the large scale flow. However, because of the need to resolve accurately the thin wall boundary layer and the wall heat transfer, the direct numerical simulations (DNS) are limited to relatively low Rayleigh number ($Ra \propto 10^7$ for the most recent computations in a fluid layer[6]). An alternative way is to resolve in time and space the large coherent eddy structures while the unresolved scales are modelled. In this way, a transient (T-) RANS approach[5] or large eddy simulations (LES)[7] [8] [9] were applied to predict the Rayleigh-Bénard flow or to validate subgrid models. These two turbulence modelling approaches are very appealed insofar as they are potentially able to reach the highest Rayleigh numbers of experimentations ($Ra \sim 10^{15}$).

The present work investigates the Rayleigh-Bénard convection flow in a fluid layer in the hard turbulent regime using large eddy simulation. The main purpose concerns the ability of the LES modelling to provide the 2/7 scaling law in order to explore eventually higher regime.

NUMERICAL SET-UP

We consider the air-flow ($Pr = 0.71$) developing in a large aspect ratio cell (larger than 4) with periodic lateral boundary conditions. The fluid is heated from below and cooled from above by two isothermal no-slip walls. The large aspect ratio cells present several advantages: a lower Rayleigh number range for hard turbulence regime and an improvement in the convergence of statistical properties[10]. The flow is solved by integrating the three-dimensional filtered Navier-Stokes equations with the Boussinesq approximation. The unknown subgrid-scale (SGS) stresses and heat fluxes, which represent the effect of the subgrid scales on the large eddies, have to be modelled. However, using an *a priori* test, it has been shown[11] that the numerical dissipation from the advective term discretization scheme (QUICK) is able to reproduce correctly the energy transfer between resolved and unresolved kinetic scales of the flow, which can be quantified by the time-averaged production of subgrid kinetic energy. Therefore, we chose not to model the SGS stresses, but only the SGS heat fluxes. We used the mixed scale diffusivity model, that we have developed and applied with success in the differentially heated cavity[11]. This original model is based on its own time-scale, so that the Reynolds analogy, which connects the SGS diffusivity with the kinetic scales through a SGS Prandtl number, is not assumed. The equations are discretized on a staggered mesh by second-order accurate finite volume approximations. The resulting system is solved by a splitting procedure with a direct solver in each direction. Incompressibility is imposed by a projection method. The code has been validated by comparison with the DNS data[6] available at $Ra = 2.10^7$. Due to the need to resolve accurately the thin wall boundary layer, we used a non-uniform mesh size in the vertical direction with grids ranging from 26×98^2 up to 82×216^2, respectively in the vertical and horizontal directions. All the simulations have been run for a time long enough to compute reliable statistics for the heat transfer and the second order moments, that require an averaging time from 300 to 100 T_L (T_L: large eddy turnover time) for the Rayleigh number $2.10^5 < Ra < 2.10^9$.

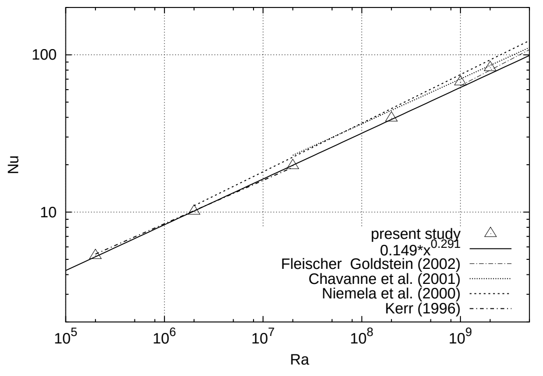

Figure 1. Comparison of the computed $Nu(Ra)$ results with several experimental and DNS correlations.

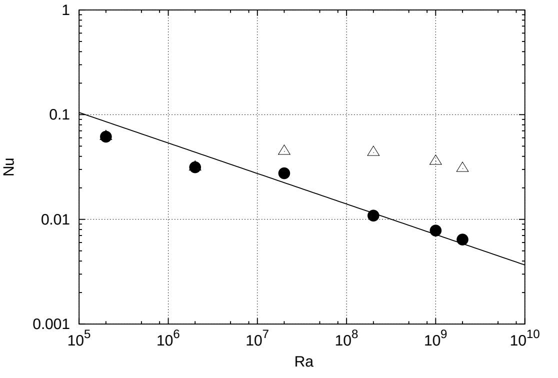

Figure 2. Viscous (△) and thermal (•) boundary layer thicknesses vs Ra. Line is power law $\delta \sim Ra^{-0.291}$.

RESULTS

In figure 1 we report the non-dimensional heat transfer as a function of Rayleigh number with some experimental results [3][1][2] for comparison, and DNS data[6] for validation, with which a good agreement is obtained. From $Ra = 2.10^5$ up to $Ra = 2.10^8$ the points seem to fit narrowly the 2/7 power law, with a best fit to our data given by $Nu = 0.149Nu^{0.291}$. Previous studies[1] have proposed different characteristics to describe the hard turbulent regime: (i) the $\sim 2/7$ power law Ra dependance for different variables, (ii) a mean large scale flow, (iii) an exponential-shaped curve for the histogram of the temperature fluctuations in the center. Now, assuming the distance from wall of the peak of the r.m.s. vertical profile as the boundary layer thickness, we observe effectively the Ra scaling law of the thermal boundary layer (Fig. 2). Moreover the histograms of the temperature fluctuations (no presented here) exhibits the exponential shaped curve for $Ra \geq 2.10^5$ instead of a Gaussian one at $Ra = 2.10^4$. Consequently, we presume to observe the hard turbulent regime for $2.10^5 \leq Ra \leq 2.10^8$.

Above $Ra = 10^9$, $Nu \sim Ra^{2/7}$ trend does not stand anymore, and a steeper curve seems to appear. This transition was also observed experimentally [1] or numerically [4] but for a higher Rayleigh number, due to their smaller aspect ratio cell. However our data present a good agreement with experimentation performed in an horizontal enclosure[3]. Besides it can be noted that a small bump is present in the decrease of the thermal boundary layer around $Ra = 2.10^7$, while the viscous boundary layer gets thicker for $2.10^7 \leq Ra \leq 2.10^9$ instead of decreasing with a constant slope. This behaviour has been also showed by previous studies and interpreted as an effect of laminar-turbulent transition of the viscous boundary layer induced by the large scale flow near the walls[1]. Further computations are needed to describe this new regime.

CONCLUSIONS

A numerical study of Rayleigh-Benard convection in an infinite fluid layer (Pr=0.71) is performed using large eddy simulation (LES) of the Navier-Stokes equations with the Boussinesq approximation. The main observation is the ability of the computations to reproduce the 2/7 scaling behavior over a large Ra range ($2.10^5 < Ra < 2.10^8$). Moreover the regime transition towards the 'ultra-hard regime' is observed at $Ra = 2.10^9$.

References

[1] X. Chavanne, F. Chilla, B. Chabaud, B. Castaing, and B. Hébral. Turbulent Rayleigh-Bénard convection in gaseous and liquid he. *Phys. Fluids*, 13(5):1300–1320, 2001.

[2] J.J. Niemela and Sreenivasan K.R. Confined turbulent convection. *J. Fluid Mech.*, 481:355–384, 2003.

[3] A.S. Fleischer and R.J. Goldstein. High-Rayleigh-number convection of pressurized gases in a horizontal enclosure. *J. Fluid Mech.*, 469:1–12, 2002.

[4] R. Verzicco and R. Camussi. Numerical experiments on strongly turbulent thermal convection ins slender cylinder cell. *J. Fluid Mech.*, 477:19–49, 2003.

[5] S. Kenjeres and K. Hanjalic. Numerical insight into flow structure in ultra-turbulent thermal convection. *Phys. Rev. E*, 66(036307):1–5, 2002.

[6] R.M. Kerr. Rayleigh numberscaling in numerical convection. *J. Fluid Mech.*, 310:139–179, 1996.

[7] T. Eidson. Numerical simulation of the turbulent Rayleigh-Bénard problem using subgrid modelling. *J. Fluid Mech.*, 158:245–268, 1985.

[8] V.C. Wong and D.K. Lilly. A comparison of two dynamic subgrid closure methods for tubulent thermal convection. *Phys. Fluids*, 6(2):1016–1023, 1994.

[9] S.J. Kimmel and J.A. Domaradzki. Large eddy simulation of rayleigh-bénard convection using subgrid scale estimation model. *Phys. Fluids*, 12(1):169–184, 2000.

[10] X.Z. Wu and A. Libchaber. Scaling relations in thermal turbulence: the aspect ratio dependence. *Phys. Rev. A*, 45:842–845, 1992.

[11] A. Sergent, P. Joubert, and P. Le Quéré. Development of a local subgrid diffusivity model for large eddy simulation of buoyancy driven flows: application to a square differentially heated cavity. *Num. Heat Transfer Part. A*, 44(8):789–810, 2003.