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OPTIMAL DESIGN OF LOSSY BANDGAP STRUCTURES

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Summary The method of topology optimization is used to design structures for wave propagation with one lossy material component. Optimized designs for scalar elastic waves are presented for mininimum wave transmission as well as for maximum wave energy dissipation. The structures that are obtained are of the 1D or 2D bandgap type depending on the objective and the material parameters.

INTRODUCTION

Elastic and acoustic (commonly referred to as phononic) bandgap structures [1] may find application for vibration suppression and for noise insulation purposes [2]. The periodic arrangement of two materials with different properties may inhibit the propagation of waves at certain frequencies and for the infinite medium total reflection of the wave occurs. So far focus has almost entirely been devoted to the wave-reflecting properties and not to the effects, possibly beneficial for applications, of dissipation in the bandgap structures. This work presents a systematic method for creating optimized designs of bandgap structures for two objectives: 1) minimum wave transmission through the structure and 2) maximum dissipation of wave energy in the structure. The structures are obtained as optimized distributions of two materials where one is lossy, and the design methodology is based on the method of topology optimization. Recently the method was used to design loss-free bandgap structures for optimized wave reflection [3].

MODEL

Wave propagation with frequency Ω is considered. The wave may be elastic or acoustic and the problem is discretized using a standard FEM formulation. The resulting field equation governing the complex wave amplitude $\mathbf{u}$ is:

$$\left(-\Omega^2\mathbf{M} + i\Omega(\mathbf{C}_{rad} + \mathbf{C}_{loss}) + \mathbf{K}\right)\mathbf{u} = \mathbf{f}, \tag{1}$$

where $\mathbf{M}$, $\mathbf{K}$ are standard mass- and stiffness-type matrices, and $\mathbf{f}$ is a forcing vector stemming from the wave excitation. The damping matrix generally comprises two terms with $\mathbf{C}_{rad}$ originating in the finite domain termination of the wave propagation, either by a simple radiation condition (Sommerfeld condition) or by extra Perfectly Matching Layers (PMLs). The matrix $\mathbf{C}_{loss}$ represents the contribution to the damping matrix from a lossy material component.

Scalar case: out-of-plane shear waves

As a special case, a 2D problem of an elastic wave propagating in the plane is investigated. By considering the out-of-plane displacement component (out-of-plane shear) the problem is reduced to a scalar one (Helmholtz equation). Two materials are defined with the relevant material properties:

$$\rho_1,\ E_1,\ \nu_1 \qquad\qquad \rho_2,\ E_2(1+i\alpha),\ \nu_2 \tag{2}$$

where ρ is the density, E Young's modulus and ν Poisson's ratio, and where α is the loss factor for the lossy material component.

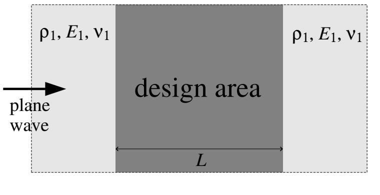

Figure 1. 2D computational domain with a scalar elastic shear wave propagating from left to right.

As an illustrative example, calculations are carried out for the 2D domain in Figure 1. A plane wave propagates from left to right through the structure containing a square design area. Unwanted reflections from the left and the right boundaries are eliminated by using PMLs to terminate the domain. Traction-free boundaries are specified on the upper and lower boundaries.

OPTIMIZATION

The goal of the optimization is to find the distribution of the two material components that minimizes (or maximizes) our objective function J. The following two objectives are considered:

$$\min J_1 = \frac{1}{nn}\sum_{n=1}^{nn}|\mathbf{u}_n|^2, \qquad \max J_2 = \frac{1}{2}\Omega^2\sum_{e=1}^{nel}\Re(\mathbf{u}_e^T\mathbf{c}_{loss}\bar{\mathbf{u}}_e) \tag{3}$$

where J_1 is the average squared amplitude of the wave transmitted through the structure and J_2 is the wave energy dissipated in the design area per unit time. In (3), nn is the number of nodal points along the right boundary where the objective is evaluated and nel is the number of elements in the design area. The element loss damping matrix is $\mathbf{c}_{loss}$, $\Re()$ denotes the real part, and $\bar{\mathbf{u}}$ is the complex conjugate field.

The optimization is performed using the method of topology optimization (see e.g. [3] for details). In the present standard FEM implementation a single design variable is introduced for each finite element in the design domain $x_e,\ e = 1, nel$. The material properties are now specified in each element as a linear function of this variable:

$$E_e = E_1 + x_e(E_2 - E_1), \qquad \rho_e = \rho_1 + x_e(\rho_2 - \rho_1), \qquad \alpha_e = x_e\alpha \tag{4}$$

so that for $x_e = 0$ the properties are those of material 1 and for $x_e = 1$ those of material 2 (the lossy material). Poisson's ratio is assumed to be constant ($\nu_1 = \nu_2 = \nu$).

The optimized material distribution is found using an iterative algorithm where the design change in each iteration is based on analytical design sensitivities and the use of a mathematical programming tool [4].

RESULTS

Figure 2a-c) show optimized designs for the minimization problem (objective J_1). The designs in a) and b) are for different loss factors ($\alpha = 0$ and $\alpha = 0.2$) and show only a small quantitative difference in the designs which are of the 2D bandgap type. The objective values show a 12.6 dB reduction in wave amplitude for a) and 14.6 dB for b). For c) the same parameters are used as in b), but a different initial guess is used in the optimization leading to a different local minimum. The design is a 1D bandgap structure (Bragg grating) with an amplitude reduction of 17.2 dB, thus making it the most efficient design obtained. Figure 2d-f) show designs for three wave frequencies for the maximization problem (objective J_2). The optimized design in d) resembles a 1D Bragg grating whereas the designs in e) and f) are more intriguing 2D structures. In all three cases the dissipation of wave energy is superior to the extreme case where the whole design area is filled with lossy material, i.e. the dissipation is increased by a factor 1.12, 1.87 and 1.50 in d), e), and f), respectively.

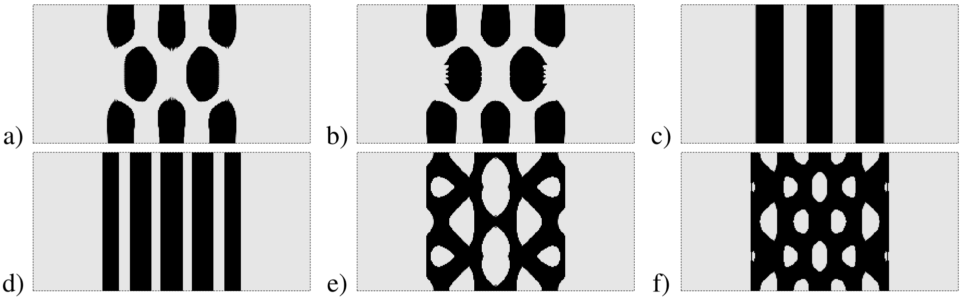

Figure 2. a-c): Minimization of wave transmission, material parameters: $E_2/E_1 = 2.5$, $\rho_2/\rho_1 = 2$, $\tilde{\Omega} = 1.5$, a) $\alpha = 0.0$, b) $\alpha = 0.2$, c) $\alpha = 0.2$ (different initial guess). d-f) Maximization of dissipation, material parameters: $E_2/E_1 = 0.5$, $\rho_2/\rho_1 = 0.8$, $\alpha = 0.05$, d) $\tilde{\Omega} = 1.5$, d) $\tilde{\Omega} = 1.75$, f) $\tilde{\Omega} = 2$. Non-dimensional frequency $\tilde{\Omega} = \Omega L/(2\pi c)$, $c = \sqrt{E_1/(2\rho_1(1+\nu))}$.

CONCLUSIONS

Optimal design of structures for wave propagation was considered. The optimized distribution of two material components, one lossy, was been obtained for two objectives: 1) minimizing the wave transmission, and 2) maximizing the dissipation of wave energy. Results were presented for elastic out-of-plane shear waves. The designs obtained are intriguing 1D and 2D bandgap type structures with good performance for both objectives. Further results will be for the cases of in-plane elastic waves, as well as optimization for a range of frequencies simultaneously.

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