

## DETERMINATION OF PHASE TRANSFORMATION YIELD SURFACE OF ANISOTROPIC SHAPE MEMORY ALLOYS

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### INTRODUCTION

The superelastic behavior of Shape Memory Alloys (SMA) is useful for several industrial applications. To determine the behavior of SMA structures, the development of specified phenomenological constitutive models is necessary. In particular, the definition of the criterion in order to detect the elastic behavior from the non-linear one related to the martensitic transformation is required. Recently, a macroscopic model, based on the concept of two transformation surfaces (a first surface drives the forward transformation and the second drives the reverse transformation) has been proposed and validated on a large data base of experimental results under uniaxial and multiaxial loadings [1-2]. This model, in its initial version, is only valid for the pseudoelastic behavior of isotropic polycrystalline SMA. However, it is well-known that the initial crystallographic texture is an important parameter in the behavior of SMA and in particular in the shape of the transformation yield surface [3-4]. The aim of this paper is to present some results concerning, on the one hand, the determination and the modeling of the transformation surface of polycrystalline textured SMA and, on the other hand, the relation between the volumic fraction of martensite and the macroscopic transformation strain.

### EXPERIMENTAL CHARACTERIZATION OF TRANSFORMATION YIELD SURFACE OF SMA

The material studied, in this part, is a Cu-Al-Be shape memory alloy. The Cu-Al-Be alloy has a weight composition of 87.75 % of Copper, 11.33 % of Aluminum, 0.49 % of Beryllium and balance trace elements including iron and silicon. After machining from wire drawn bars, all Cu-Al-Be specimens were treated at 650°C for 20 minutes in atmospheric air and quenched in boiling water at 100°C for 1 hour. The four transformation temperatures at stress free state have been determined by using electrical resistance measurements, (the austenite start temperature,  $A_s^0=19^\circ\text{C}$ , the austenite finish temperature,  $A_f^0=32^\circ\text{C}$ , the martensite start temperature,  $M_s^0=21^\circ\text{C}$ , and the martensite finish temperature,  $M_f^0=-7^\circ\text{C}$ ). A series of tests under tension (compression)-internal pressure and tension (compression)-torsion on tubular specimen and under bi-axial compression on cubic specimen has been performed to determine the initial surface of phase transformation of Cu-Al-Be SMAs in the principal stress plane  $(\sigma_1, \sigma_2)$  and in the axial-shear stress plane  $(\sigma, \tau)$ . Figures 1 and 2 show the experimental results.

### MACROSCOPIC CRITERION OF ONSET TRANSFORMATION FOR ANISOTROPIC SMA

This part is concerned with a proposition of a generalized macroscopic criterion of transformation onset based on the experimental results presented herein. The main objective of this macroscopic criterion is the description of the boundary of the domain, in the stress space, in which one the martensite phase transformation is not activated. This macroscopic criterion has the same role as the yield domain in the case of elasto-plastic materials. One can assume that the martensite transformation of SMAs is volume-invariant. Therefore the criterion for transformation onset,  $f$ , can be supposed to be independent of the first stress invariant,  $\bar{P}$ . Then, the following choice is made:

$$f(\bar{\sigma}^H, y_{ij}, \sigma_0) = \bar{\sigma}^H g(y_{ij}) - \sigma_0 \geq 0 \quad \text{with} \quad g(y_{ij}) = \cos \left[ \frac{\cos^{-1} \left( 1 - a(1 - \sqrt{y_{ij}}) \right)}{3} \right]$$

where  $\bar{\sigma}^H = \sqrt{H \text{dev}(\boldsymbol{\sigma}) : \text{dev}(\boldsymbol{\sigma})}$  is the Hill's equivalent stress,  $\sigma_0$  is the tension yield stress and  $y_0$  is the third stress invariant. H and  $y_0$  are defined by the following relations:

$$H = \frac{3}{2} \begin{bmatrix} c & 0 & 0 & 0 & 0 & 0 \\ 0 & d & 0 & 0 & 0 & 0 \\ 0 & 0 & d & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad y_0 = \frac{27}{2} \frac{\det(\text{dev}(\boldsymbol{\sigma}))}{\left[ \sqrt{\frac{3}{2} \text{dev}(\boldsymbol{\sigma}) : \text{dev}(\boldsymbol{\sigma})} \right]^3}$$

The values of the material parameters have been identified with the experimental results. Figures 1 and 2 show the comparison between the experimental results and the macroscopic criterion. The macroscopic criterion is in good agreement with the experimental observations. Moreover, Figure 3 shows that the initial transformation strain rate is normal to the initial surface of martensite transformation.

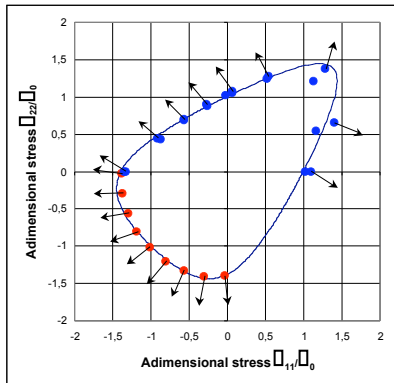


Figure 1. Critical stresses at the onset of phase transformation in  $(\sigma_1, \sigma_2)$  for polycrystalline Cu-Al-Be

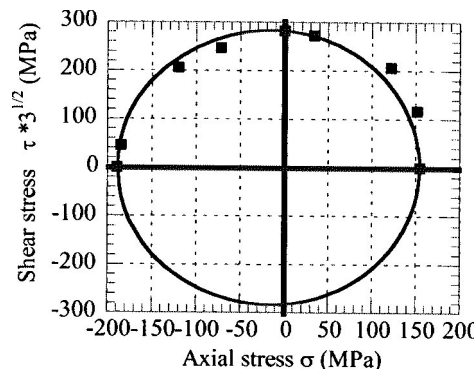


Figure 2. Critical stresses at the onset of phase transformation in  $(\sigma, \tau)$  for polycrystalline Cu-Al-Be

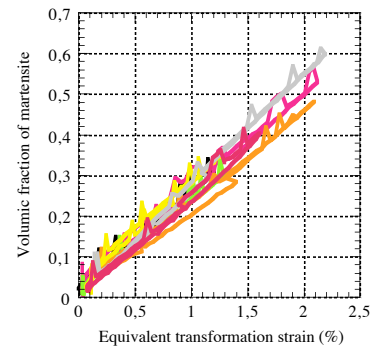


Figure 3. Volumic fraction of martensite versus equivalent transformation strain for different prop. tension-torsion loadings

**RELATION between MARTENSITE VOLUMIC FRACTION and TRANSFORMATION STRAIN**

A great number of authors has proposed a linear empirical relation between the martensite volumic fraction,  $f$ , and the transformation strain,  $\epsilon^r$ . This relation has been identified only under uniaxial tension-compression loading [5]. We propose in this section a generalization of this relation for any multiaxial proportional loadings. One can show that a general equivalent strain,  $\epsilon_{eq}^r$ , can be built and given by the following relation:

$$\epsilon_{eq}^r = \frac{\bar{\epsilon}^H}{\sqrt{G(y_0, a, H)}}$$

where  $G$  is a function,  $\bar{\epsilon}^H$  is the Hill equivalent transformation strain and  $y_0$  is the third strain invariant. A series of tests under tension (compression)-torsion on tubular specimen has been performed to determine the relation between the volumic fraction of martensite and the equivalent transformation strain. During the tests, the volumic fraction of martensite has been determined by using electrical resistance measurements and the equivalent transformation strain has been calculated with the strain gauges measurements. Figure 3 shows the experimental results for different loading paths on a Cu-Al-Be SMA. We can see that the linear relation between volumic fraction of martensite,  $f$ , and the equivalent transformation strain,  $\epsilon_{eq}^r$ , is validated.

**References**

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