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MODELLING FATIGUE CRACK GROWTH WITH TIME-DERIVATIVE EQUATIONS.

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Summary Fatigue crack growth is difficult to predict in case of complex loadings because of load history effects. A model based on a set of time-derivative equations has been developed within the framework of the thermodynamics of dissipative processes. It consists in a crack propagation law and a constitutive behaviour law for the cracked structure determined from finite elements analyses. Capabilities of the model to reproduce typical fatigue results have been tested and proved.

INTRODUCTION

Engineering models struggle with loads-load interaction effects in fatigue crack growth. Monotonic fatigue crack growth rates are usually successfully predicted using the Paris law. Under non-monotonic fatigue, damage accumulation rules are employed. The crack extension after a given lifetime is calculated by summing the contributions to the crack extent of each elementary fatigue cycle Ni, inside a non-monotonic loading sequence.

$$a(N+1)=a(N)+\left[\frac{da}{dN}\right]_{N_i}=a(N)+C.\left(\Delta K_{eff}^{i}\right)^{m}=a(N)+C.\left(K_{\max}^{i}-K_{op}^{i}(history)\right)^{m}$$

The crack opening stress intensity factor (K_{op}) can be introduced at this stage, in order to account for history effects inherited from crack tip plasticity [1]. Various methods are employed to determine the evolution of K_{op} with load history. It was shown previously that the sensitivity of K_{op} to load history is related to the sensitivity of the material itself to load history, which can be described using a constitutive behavior for the material including a kinematics and an isotropic hardening [2]. Provided that suitable constitutive equations are employed for the material, the finite element method is successful in predicting the evolution of K_{op} with load history and the evolution of the crack growth rate under a cyclic but non-monotonic fatigue loading.

However, at this stage there is no getting away from a fundamental difficulty of such approaches. As a matter of fact, the Paris law is a cycle-derivative equation. Therefore, before applying any damage accumulation rule, elementary cycles should be extracted from a loading sequence which may not be cyclic. For this purpose, counting methods are employed, such as, for instance, the “rainflow” method. With that method, small amplitude “cycles” are embedded inside high amplitude “cycles”. However, in order to apply the damage accumulation rule, elementary cycles have to be applied consecutively, which imposes to make a choice. For any given portion of a complex signal, various combinations are always possible. The selection of the most conservative configuration is difficult because it depends on load history.

To sum up in a few words, because the Paris law is a cycle-derivative equation, cycles have to be extracted from real signals. However, because of load history effects this operation is questionable. Therefore, if we aim at taking into account load history effects, this operation should be avoided because it may modify significantly the load history. Therefore, a set of time-derivative equations was searched for fatigue crack growth, including load history effect inherited from crack tip plasticity.

BACKGROUND OF THE MODEL

The model was developed on the basis of the thermodynamics of dissipative processes. Three global variables are employed to characterize the velocity field at crack tip, the crack growth rate da/dt, the rate of plastic blunting dρ/dt and the rate of elastic crack opening. Thermodynamics counterparts are introduced for each velocity variable. Since crack tip residual stresses are well known to considerably influence fatigue crack growth under variable amplitude loading, the elastic energy stored inside the crack tip plastic zone is included in the energy balance equation. This equation leads to establish a yield criterion for the cracked structure. In this criterion, the energy stored within the crack tip plastic zone (residual stresses) leads to the appearance of a kinematics hardening term for the cracked structure. The second principle of thermodynamics provides an inequality between da/dt and dρ/dt, that the instantaneous cracking law employed in the model should observe. Finally, in the present model, the following crack propagation law was chosen, in agreement with the second law:

$$\frac{da}{dt}=\frac{\alpha}{2}\left\langle -\frac{d\rho}{dt}\right\rangle \quad with \begin{cases} \frac{d\rho}{dt}\geq 0 & \left\langle -\frac{d\rho}{dt}\right\rangle = 0 \\ \frac{d\rho}{dt}<0 & \left\langle -\frac{d\rho}{dt}\right\rangle = -\frac{d\rho}{dt} \end{cases}$$

Once this propagation law is set, the crack growth rate is determined from the crack tip blunting rate. Therefore, the evolution law of crack tip blunting is still needed. The evolution of crack tip blunting during the loading sequence, is merely a mechanical problem and can be determined using the finite element method.

In this purpose, the conjugate force ϕ as opposed to crack tip blunting has been considered. When there is dissipation, this force is the sum of a conjugate force as opposed to plastic deformation within the crack tip plastic zone ϕ_{eff} and of a conjugate force as opposed to the storage of elastic energy within the crack tip plastic zone ϕ_X.

$$\phi=\phi^{eff}+\phi^{X}$$

This equality should always be observed when there is dissipation. Therefore this equation is a yield criterion for the cracked structure. The evolution equations of ϕ_{eff} and of ϕ_X with respect to an increase of the crack length or to a variation of crack tip blunting are still needed. These equations are the constitutive equations for the cyclic plasticity of the cracked structure.

From the finite elements calculations, it appeared that there are two distinct thresholds for the plasticity of the cracked structure. The first one corresponds to the plasticity within the cyclic plastic zone and the second one to the extent of the monotonic plastic zone. The “cyclic” threshold is reached first, and is let aside as soon as the “monotonic” threshold is activated. In such a case, four internal variables are now necessary :ϕ_m^{eff} and ϕ_c^{eff} respectively opposed to plastic deformation within the monotonic and the cyclic plastic zone, ϕ_m^X and ϕ_c^X respectively opposed to the storage of elastic energy within the monotonic and the cyclic plastic zone.

Various types of loading schemes were employed to determine the partial derivative of each of these four internal variables with respect to an increase of the crack length, and with respect to a variation of crack tip blunting. A set of 8 partial equations has been found with the involvement of 10 parameters. Except for the parameter α in the crack propagation law, the material parameter are determined using the finite element method and depend on the specific mechanical behaviour of the material.

The model was also implemented and tested. It reproduces successfully the Paris law under monotonic fatigue (fig.1). It allows also reproducing well known effects in non-monotonic fatigue, such as the stress ratio effect and the overload retardation effect.

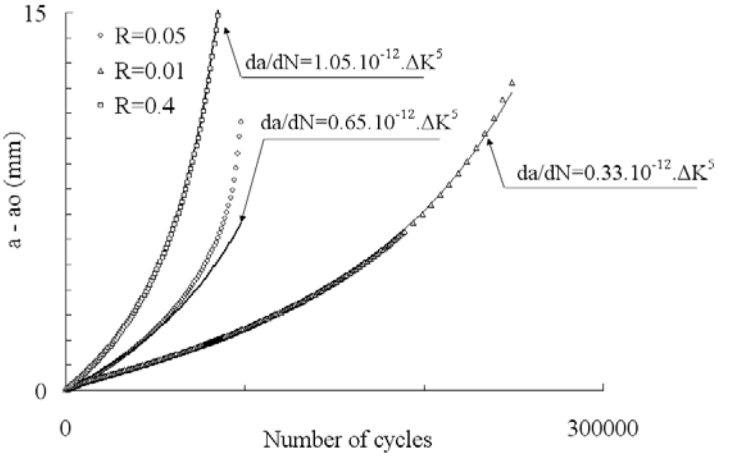

Fig. 1: Crack extent (a_o=10 mm) as predicted using the present model under a sinusoidal signal with a stress ratio equal to 0.01, 0.05 or 0.4.

CONCLUSION AND PERSPECTIVES

This approach appears to be promising to deal with the crack propagation previsions under variable amplitude loadings. It allows to free from the cycles extraction techniques by following directly the crack advance as a function of time. Moreover, it is expected to deal with the coupling of fatigue phenomena with time-dependent mechanisms such as oxidation and creep. The experimental validation of the model remains to be done by confronting numerical evolutions with crack propagation measurements. The extension of the model to multiaxial loadings is also planned.

References

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[2] Pommier S., Cyclic plasticity and variable amplitude fatigue *Int.. Jour. Fat.* **25**:983-997, 2003.