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MODIFIED ERROR IN CONSTITUTIVE RELATION AND ITS APPLICATION TO DYNAMIC TESTS WITH CORRUPTED BOUNDARY CONDITIONS

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Summary The aim of this work is to develop an identification strategy suitable for the identification of models dedicated to crash simulation. Due to the experimental difficulties, the method has to take into account the uncertainties on the measurements. Therefore, a method is proposed, based on the modified error in constitutive relation. First developed in the case of homogeneous elasticity, this method appears to be very robust and is extended to the case of heterogeneous elasticity.

INTRODUCTION

The purpose of this study is the prediction of the crash behavior of multi-layered composites. The simulation of tests performed on such absorbing materials involves several difficulties:
(1) the insufficient knowledge of the dynamic behavior and crashworthiness of the materials involved,
(2) the often imperfectly known boundary conditions,
(3) the heterogenous state implied by damage and rupture localization.
The dynamic tests we are considering are dynamic compression tests on laminated plates based on Split Hopkinson Pressure Bar principle. These lead to imprecise measurements of forces and velocities. Thus, Problem (2) is encountered from the very beginning of the identification process. In a first attempt, we have made use of a method proposed in [1] which is based on a comparison between two calculations, one with prescribed displacements and one with prescribed forces. This method was proved to be very efficient for well mastered dynamic tests but appears to be unadapted in the case of strong uncertainties on the boundary conditions. The problem that we have addressed is therefore the following:
How can one formulate an identification problem in dynamics in such a way that the results would be accurate in spite of the great measurement uncertainties?

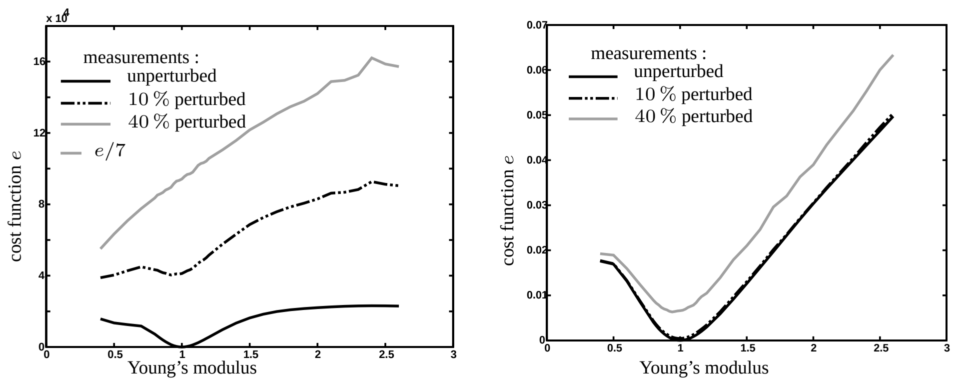
IDENTIFICATION STRATEGY

Our guiding principle, which was directly inspired by studies on model updating in vibration [2], is to verify exactly, during the identification process, the properties which are considered to be reliable. The uncertain quantities are then taken into account by minimizing a modified constitutive relation error [3]. In this first work [4, 5], we are considering the simple problem of an elastic rod with redundant displacement and force conditions at both ends, denoted $\tilde{u}_d$ and $\tilde{f}_d$, which are assumed to be uncertain. The identification of the Young's modulus E is carried out in two steps: for a fixed E, the ill-posed problem is reformulated as the minimization of:

$$e(u, \sigma, u_d, f_d) = \| \sigma - E.\epsilon \|^2 + \alpha \| f_d - \tilde{f}_d \|^2 + \beta \| u_d - \tilde{u}_d \|^2 \qquad (1)$$

under the conditions:

$$u \text{ KA to } u_d, \;\; \sigma \text{ DA to } f_d, \;\; \rho.\ddot{u} - div\sigma = 0 \qquad (2)$$



(a) classical approach

(b) our method

Figure 1. Variation of the error with E for different perturbation levels

One can note that the boundary conditions u_d and f_d are related to the fields used in the minimization problem and, therefore, can be different from the measured values $\tilde{u}_d$ and $\tilde{f}_d$. This minimization under constraints is performed by introducing an appropriate number of Lagrange multipliers, which leads to the simultaneous resolution of both a direct problem and an adjoint (time-retrograde) problem related to one another. The resolution methods will be discussed during the presentation. The regularization properties of the method will be discussed along with the influence of the weights α and β between the different terms of the error.

Subsequently, the identification of the best E is carried out using the same functional, whose gradient can be calculated directly from the fields which are solutions to the first problem with fixed E. A numerical example, in which the exact measurements were modified by up to 40% on the average, is presented figure 1. The method proposed here, which turns out to be particularly robust, and the one inspired by [1] are compared.

Then, the robustness of the method is studied by introducing two other formulations deduced from the previous one by removing the distance to the measurements in one or two steps of the method. The comparison of the three methods leads to the conclusion that the method is efficient with respect to the perturbations, because it takes the whole experimental information in one calculation and it introduces a distance between the measurements and the calculation.

EXAMPLES IN HETEROGENEOUS CASES

The results of the proposed method in term of robustness led us to extend the method to more complicated cases. The first one, illustrated 2, is the identification of the properties of a heterogeneous beam, where the spatial distribution of the properties is known. Other examples such as the identification of the size of heterogeneities are treated.

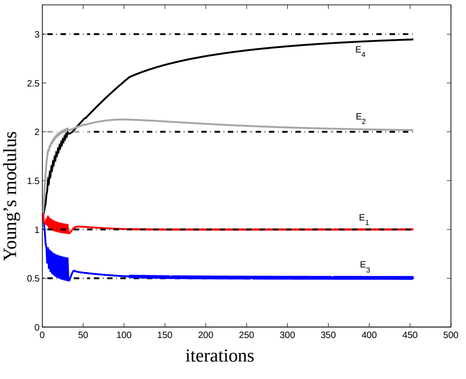


Figure 2. Young's modulus as a function of the iterations

CONCLUSION

The proposed method appears to be very robust with respect to the perturbations on the measurements. Its robustness is due to the taking into account of the whole experimental information in one calculation and the introduction of a distance between the experimental data and the calculation. A first step toward the identification of models suitable for crash simulation has been made by treating examples in heterogeneous elasticity, where the method remains accurate.

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