# CHAOTIC RESPONSE OF NON-REVERSIBLE DRY FRICTION OSCILLATOR

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*Summary* In this short communication we investigate how the non-reversible dry friction characteristics will alter the non-linear responses of a simple mechanical oscillator. The presented approach is based on a certain mathematical description of the experimentally determined non-reversible dry friction characteristics, which causes chaotic and irregular motion of the studied system. A novelty of our idea is an introduction of the relative acceleration in description of the dry friction model.

## INTRODUCTION

For many years the topic of dry friction has been actively researched with many attempts to identify the causes of unwanted behavior such as squeal of car brakes, extensive wear of the cutting tools, and others. A practical engineering approach, indebted to Coulomb simplifies the friction force to constant value directed opposite to the relative velocity of the contacting bodies. Such force can take two values with identical level but opposite sign only. Newer experiments show non-linear dependence on the contact velocity rather than the constant one. That was why most efforts were directed to built non-linear friction models and to determine differences in maximal values of the static and dynamic friction forces – see [3]. Another attempts to determine different types of friction characteristics showing dependencies on the relative acceleration on the contacting surface are so-called non-reversible friction characteristics [1,2,5]. However, from a viewpoint of the experiment, even such non-linear approaches well approximate real character of friction force only for periodic responses of the considered system. In case of irregular, non-periodic motion of a non-linear friction oscillator, friction characteristics generated from experiments, often have a form of area filling curves, in co-ordinates friction force versus relative velocity [4]. This fact cannot be explained by measurement errors and rises a need to formulate a more general, universal friction model taking under consideration also the irregular motion of the system with dry friction.

## DRY FRICTION MODELLING

We will use the experimental data obtained from the dry friction oscillator, which allows to induce conditions where the relative velocity changes its sign. There is a good deal of flexibility in varying frequency and amplitude of excitation, and combination of frictional materials. In this study we will be using the cleanest data which comes from experiments steel on teflon. To avoid the double counting of viscous forces, in Figure 1, the viscous component in each dry friction force-relative velocity loop has been subtracted. The above described dependence between friction characteristic reconstructed experimentally and a kind of observed motion has been induced us to new approach to the problem of dry friction modelling. According to our intention such a developed model of dry friction should have a universal character e.g. it well approximates real relation between friction force and relative velocity in case of irregular motion of the system and it simplifies to a form similar to known friction characteristics when motion is regular. This condition is fulfilled by the example of dry friction model presented below which has been formulated on the basis of non-reversible friction model [5]. The proposed novel model has a symmetrical non-reversible characteristics (Eq.(1a)), with an auxiliary assumption that the parameter in the exponent is a function of the relative acceleration (Eq.(1b)) in contrast to primary model, where this coefficient is constant. Hence, the model is described with the following equations:

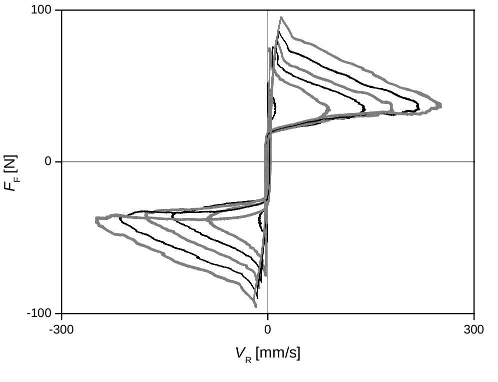


Figure 1. Family of dry friction characteristics

$$f = \begin{cases} f_u, & \operatorname{sgn}(\ddot{x}) > 0 \\ f_l, & \operatorname{sgn}(\ddot{x}) < 0 \end{cases} \quad \rightarrow \quad \begin{aligned} f_u &= f_d\left[1 + \frac{f_s - f_d}{f_d}\exp\left(-a(\ddot{x})|\dot{x}|\right)\right] \\ f_l &= f_d\left[1 - \frac{f_s - f_d}{f_d}\exp\left(-a(\ddot{x})|\dot{x}|\right)\right] \end{aligned} \tag{1a}$$

$$a(\ddot{x}) = \frac{a_1}{|\ddot{x}| + a_2} \tag{1b}$$

where $a_1$, $a_2$ >0 are constant parameters $f_s$, $f_d$ – static and dynamic friction coefficient respectively.

To show universal character of the above presented model let us consider one-degree-of-freedom, mechanical oscillator to which a dry friction damper has been added where the model under consideration (Eqs(1a) and (1b)) has been applied in numerical simulations. The dynamics of this system can be described by a simple non-smooth first order differential equations

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= q\cos(\omega t) - \alpha g(x_1) - 2hx_2 - \varepsilon f sign(x_2), \end{aligned} \tag{2}$$

Universal nature of proposed method of dry friction modelling is presented in Figs 2a and b, where friction characteristics for two different types of the oscillator under consideration and respective phase portraits of these systems are shown. It is clearly presented that for linear oscillator having periodic solution (Fig.2a, $g(x)=x$), friction model possesses a shape predicted in mathematical description of the classical non-reversible model and contains two lines representing relative the acceleration and deceleration phases respectively. In case of non-linear oscillator with chaotic attractor (Fig. 2b, $g(x)=x^3$) friction characteristic is an transformed image of this attractor according to the hypothesis presented above.

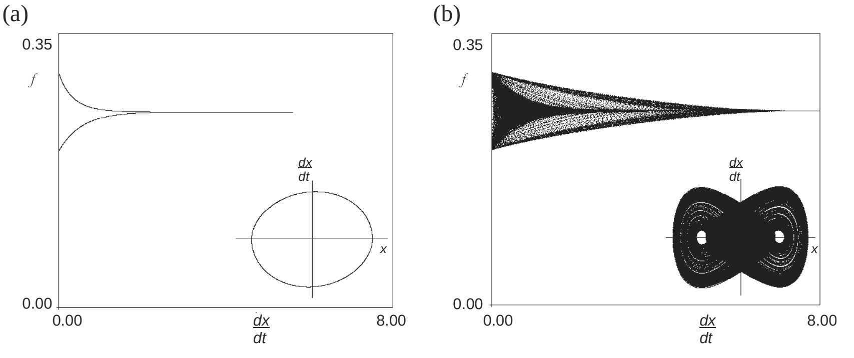


Figure 2. Friction characteristics for the novel friction model and the respective phase portraits of the oscillator applied in numerical experiment for two cases: a) linear friction oscillator– $g(x)=x$, $\alpha$=1.00, $h$=0.05, $q$=0.50, $\eta$=1.00, $\varepsilon$=0.5, b) non-linear friction oscillator – $g(x)=x^3$, $\alpha$=1.00, $h$=0.05, $q$=10.00, $\eta$=1.00, $\varepsilon$=0.5; parameters of the friction model (Eqs.1): $f_s$=0.30, $f_d$=0.25, $a_1$=12, $a_2$=0.10.

## CONCLUSIONS

The presented model of dry friction only exemplifies the way of friction modelling and it is only the first step to formulate a novel model of dry friction for non-linear systems. Earlier numerical experiments carried out by authors have shown, that introduction of this model represented by Eqs(1), does not results in qualitative changes of motion character in comparison with another friction models. Hence, from a viewpoint of engineering applications, where the dynamics is simple, it is sufficient to apply classical Coulomb law. However, our recent experiments show that even for our oscillator where $g(x)$ is linear, the system can respond with an irregular stick-slip motion. Therefore there is a need for more comprehensive dry friction models, and this paper outlined one.

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