Coherent structure of point vortices influenced by uniform straining flow

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Summary

Two-dimentional numerical simulations of motion of an elliptic vortex patch modelled by a group of identical point vortices in an inviscid uniform straining flow is described. The behaviour of the patch as a function of strain rate in the external flow is presented. It is shown that for strong strain the cloud may come apart or survive as a remnant of the initial one. Moreover, we observe disturbances created at the edge of the patch that penetrate into the structure and destroy its internal order.

Introduction

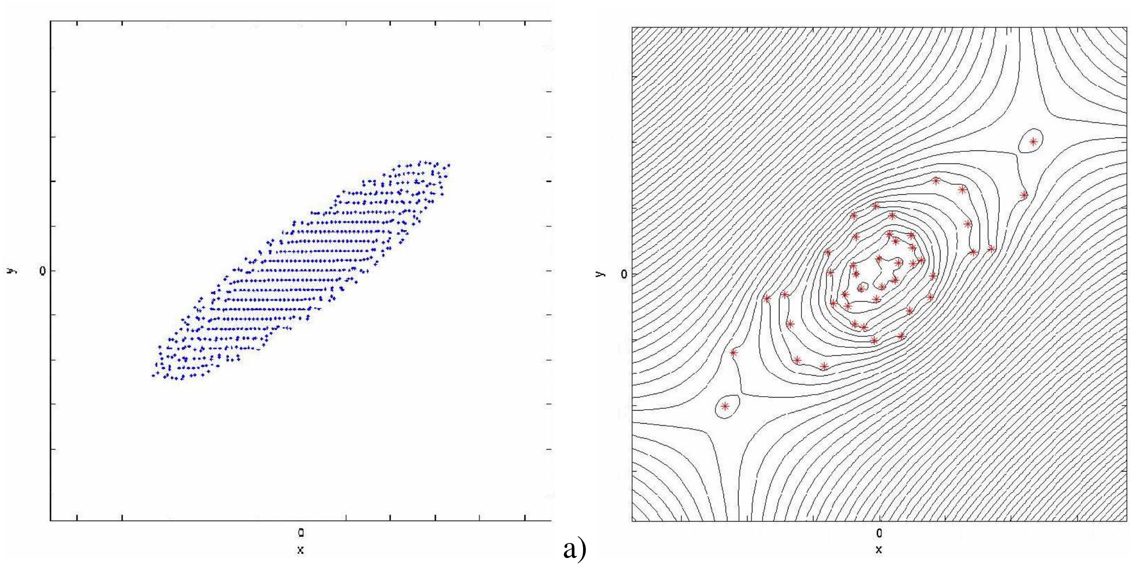
The theory of point vortices is well developed [1]. When a two-dimensional straining flow is applied it still remains a Hamiltonian system. For a coherent structure in an external flow dispersion is not an invariant of motion, so there are no restrictions on the positions of the vortices, and the patch can decay. We simulate it by the method of point vortices which also gives us the information about the internal processes. The external strain models the influence of other distant vortices. It is stronger when the distant vortices are closer to the patch.

Results of the numerical experiments

The initial condition is a homogeneous elliptical structure of identical point vortices, which represents a patch of a constant vorticity. The cloud of vortices in a straining flow rotates around its center of vorticity and its ellipticity oscillates. The stronger flow increases the amplitude of oscillations. It also changes an angular velocity of the patch into non-uniform. The internal structure, unlike to its shape, rotates with constant angular velocity, independently on a strain. Disturbances created at the edge of the patch penetrate into the structure and gradually destroy its internal order (fig. a). Density of point vortices within the patch becomes non-uniform. The stronger flow causes a ”mixing” process more effective and a chaos expands faster. Typical forms of heterogeneity are little groups, lines and pairs of vortices rotating around the local centers of vorticity. Fluctuations of density strongly depend on a strain rate and can trigger loss of small groups of vortices. That situation in terms of streamlines is presented in a fig. b. The decay of the cloud can proceed slowly (single vortices are being ”scraped” by the flow) or rapidly (vortices leave the structure and the flow can penetrate the cloud deeper to pull out more vortices). The elliptic shape of the cloud can change into a spiral. At a critical strain the axes are rotated by $\pi/4$ and two little streaks of vortices flow out from the cloud (quasi-steady state) (fig. c). It is possible that fluctuations of density can stop the leakage of vortices and the structure can survive as a remnant of the initial one. As a consequence of the Kelvin-Helmholtz instability the cloud comes apart and a big structure turns into a vortex street (fig d).

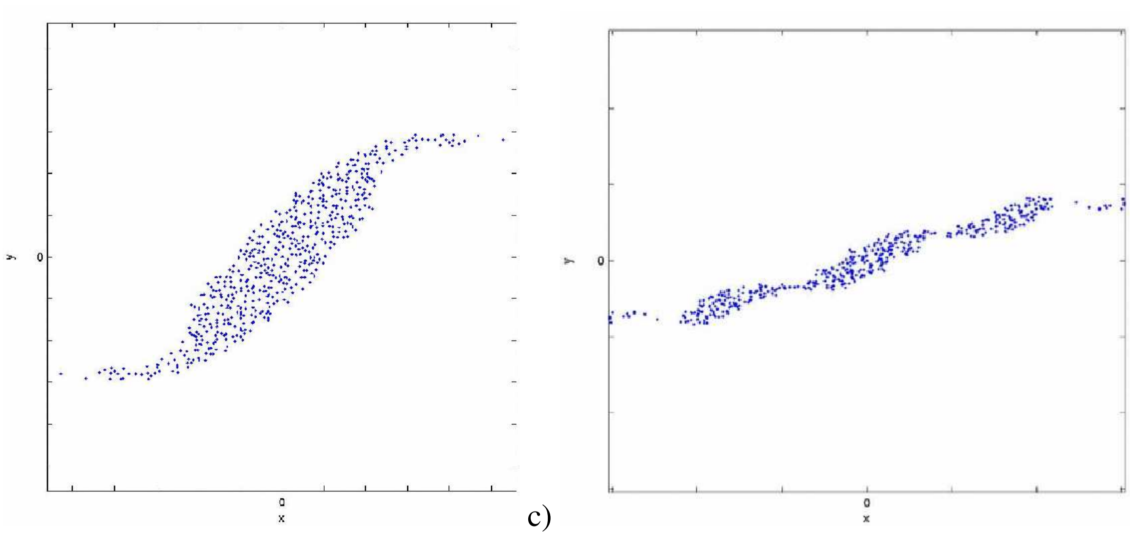
Conclusions

Two-dimensional numerical analysis of interactions between coherent structure of point vortices and an external straining flow is presented. At the first approximation that flow is the influence of other distant vortices, so we can treat it as an example of interactions between



a) b)

a) Disturbances formed at the edge of the patch penetrating into the structure.
b) Single vortices pulled out by the flow (lines are temporary streamlines).



c) d)

c) Leakage of vortices through the "gates" of two main stagnation points.
d) Remnant of an initial elliptical structure as a vortex street.

vortices. The external strain causes the behaviour of the patch more irregular and the stronger one increases fluctuations of density and a mixing time (see above). Close to the critical value the patch can lose vortices and finally transform into a vortex street.

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