INVESTIGATION OF COUPLE-STRESS EFFECTS IN ELASTIC BODIES UNDER DEFORMATION

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Summary The paper presents new analytical and numerical solutions to some problems of asymmetric elasticity theory. A comparison is made with the analytical solutions obtained in the framework of the classical elasticity theory. The results of experimental investigation of the couple-stress effects in materials under deformation are discussed.

INTRODUCTION

At present, in the framework of solid mechanics a number of models have been developed, in which the deformation of a medium is described not only in terms of the displacement vector but also in terms of the rotation vector. The classical works on the asymmetric elasticity theory state that the versions of this theory will be able to provide a better agreement with experiments, in which a material is subjected to essential gradient stresses. However, the engineering applications of these models are practically absent because there are no reliable data on the material constants of asymmetric elasticity theory. Moreover no experiments have been conducted to determine couple–stress effects in elastic solids under deformation.

One of the key factors that could provide an objective estimation of the usefulness of the asymmetric elasticity theory and offer considerable scope for experimental investigation is a sufficient amount of problems solved in the framework of this theory.

ANALYTICAL SOLUTIONS

The work presents a number of new analytical solutions to the problems of asymmetric elasticity theory [1]: shear deformation of a plane infinite layer (plate) fixed at both edges under the action of gravitational force (Fig.1.a); torsion deformation of a ring rigidly fixed at the external contour due to rotation of the internal contour by a fixed angle (Fig.1.b); deformation of a ring with a rigidly fixed external contour due to displacement of the internal contour by a prescribed value (Fig.1.c); the Kirsh problem on the uniaxial tension of an infinite plate weakened by a central circular hole (Fig.1.d).

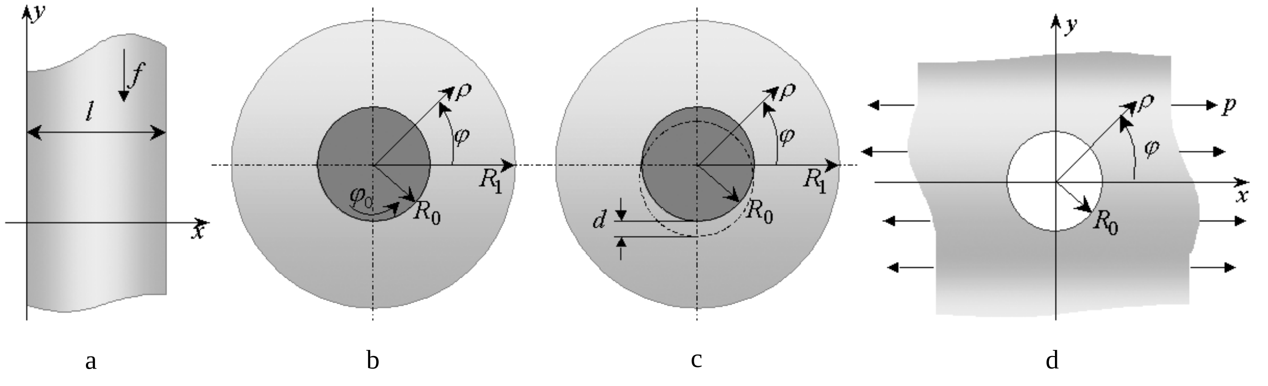
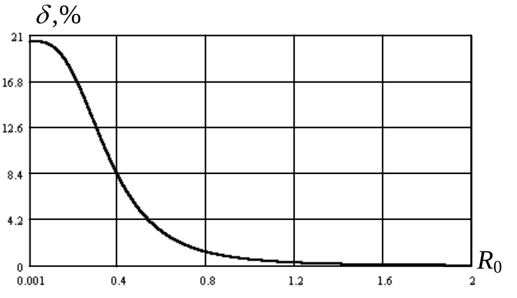


Fig.1.

By comparing the dimensionless forms of the obtained solutions with their classical analogues we can draw a conclusion that solutions of the asymmetric elasticity theory are inhomogeneous functions of the characteristic geometrical dimension, whereas classical solutions are homogeneous functions. The obtained solutions have been analyzed with reference to macro-quantities that can be measured by experiment. For the first problem these are maximal axial displacements. For the second problem this is the torque moment at the internal contour, which is defined by the value of the internal contour stress. For the third problem this is the axial force of the internal contour response also defined by the value of stress on the internal contour. In the fourth problem the characteristic macro-parameter is the degree of distortion of the hole contour.

For the problems under consideration the above mentioned macro-quantities determined in terms of the asymmetric elasticity theory were compared with those obtained by the classical theory. The material used to obtain quantitative results was synthetic polyurethane, whose material constants are presented in work [2]. An example of such a comparison is given in Fig.2.



$D = \left| \dfrac{u_\rho(R_0, 0)}{u_\rho(R_0, \pi/2)} \right|$ – degree of distortion of a circular hole;

$\delta = \left| \dfrac{D - D^*}{D^*} \right| \cdot 100\%$,

D – couple-stress solution
D^* – classical solution.

Fig.2.

NUMERICAL SOLUTIONS

To extend the scope of solvable problems we have developed an algorithm of the finite element method, which apply well to the problems of asymmetric elasticity theory. The finite element realization is based on the following variational equation of the asymmetric elasticity theory:

$$\int_V \left(\tilde{\sigma}\cdot\cdot\delta\tilde{\gamma} + \tilde{\mu}\cdot\cdot\delta\tilde{\chi}\right)dV - \int_V \left(\vec{X}\cdot\delta\vec{u} + \vec{Y}\cdot\delta\vec{\omega}\right)dV = \int_S \left(\vec{p}\cdot\delta\vec{u} + \vec{m}\cdot\delta\vec{\omega}\right)dS \qquad (1)$$

where: $\vec{u}$, $\vec{\omega}$ are the displacements and rotation vectors; $\tilde{\sigma}$, $\tilde{\mu}$ are tensors of force and couple stresses; $\tilde{\gamma}$, $\tilde{\chi}$ are the strain and bending-torsion tensors; $\vec{p}$, $\vec{m}$ are the vectors of forces and couple stresses prescribed on the boundary; $\vec{X}$, $\vec{Y}$ are the vectors of mass forces and mass moments.

The finite element used in calculations is a triangle with quadratic approximation of the displacement vector components and linear approximation of the rotation vector components. The developed finite-element algorithm has been used to obtain numerical solutions to a number of problems, in which couple-stress effects are more pronounced than in the problems for which the analytical solutions have been derived. For example, the problem of a plate weakened by several circular holes under tensile load.

EXPERIMENTAL INVESTIGATION

The experimental scheme of material deformation was developed in the context of the Kirsh problem. To define the appropriate dimensions of the plate we performed numerical calculations, which allowed us to eliminate the influence of the plate dimensions on the stress state in the vicinity of the central circular hole.

Two experimental schemes were realized. The first scheme was intended to measure the degree of distortion of the hole contour during extension of the plate. The second scheme involved measurements of the variations in the plate thickness along its contour. The surface profile was measured using the “New View-5000” microscope-interferometer. As a result of experiments the dimensions of the hole, at which the material under deformation exhibited couple-stress effects, were defined.

CONCLUSION

In this work, four new analytical solutions to the problems of asymmetric elasticity theory have been constructed. The obtained solutions allow us to identify the macroscopic parameters, which can be measured by available methods during experimental realization of the appropriate problems. To demonstrate the effects of couple-stress behavior of the material the obtained solutions have been compared with the corresponding solutions of the classical elasticity theory. The finite-element algorithm has been developed for solving two-dimensional problems of asymmetric elasticity theory. The algorithm enables us to extend the scope of the problems, which can be effectively used to analyze the couple-stress behavior of the materials. The experimental investigations have been performed to reveal situations, in which solids under deformation exhibit couple-stress effects.

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