NONLINEAR STABILITY OF ROTATING CHANNEL FLOW

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Summary The stability of Poiseuille flow in a channel subject to a system rotation about a spanwise axis is considered. Linear stability results show that the basic flow first loses stability to a two-dimensional streamwise-independent disturbance. We find two-dimensional nonlinear secondary flows, and analyse their stability. A rich structure of relationships exists between the secondary flows which does not exist for the equivalent Couette problem. We proceed to find three-dimensional nonlinear travelling-wave tertiary flows which bifurcate from the secondary flows. Three distinct three-dimensional tertiary flows are identified including flows which resemble the twisting vortex flows observed in previous experimental and DNS studies.

INTRODUCTION

The stability of channel flow subject to system rotation is of relevance to many geophysical and astrophysical problems, as well as having many engineering applications, for example coolant flow in turbine blades or the flow inside impellers of centrifugal pumps. The present study investigates channel flow subject to system rotation about an axis in the spanwise direction. The stability of this flow is determined by two dimensionless parameters; the rotation number, Ω, and Reynolds number, R. Linear stability analysis [3] predicts a minimum critical Reynolds number, R_c^{ro}, of many orders of magnitude less than the corresponding value for plane Poiseuille flow. This result has been confirmed experimentally by Alfredsson & Persson [1], who found the setting up of streamwise vortices when $R > R_c^{ro}$ and Ω lies in a certain range. At higher values of R, as Ω is increased a secondary instability occurs which leads to an adjustment of the spanwise wavenumber, while at higher R a three-dimensional travelling-wave tertiary flow may occur [1].

BASIC FLOW AND LINEAR STABILITY

The governing equations, the Navier-Stokes equations together with the incompressibility condition, are expressed in a rotating frame of reference. Seeking a steady basic solution whose only non-zero velocity component lies in the streamwise direction, we derive a parabolic basic-velocity profile together with a pressure gradient in the wall-normal direction. Consideration of the linear stability of this basic flow results in a linear differential eigenvalue problem, whose solution was obtained using a Chebyshev collocation-point numerical method. We find good agreement with the well-known results of previous authors for the non-rotating case, and the existing results for the rotating case[1, 3]. For the rotating case, the flow first becomes unstable to a two-dimensional streamwise-independent mode at the critical Reynolds number $R_c^{ro} = 66.447$ (for R based on maximum speed and half channel width). For larger Reynolds numbers the basic flow may also lose stability to disturbances of non-zero streamwise wavenumber, α. However, for a given spanwise wavenumber, β, the mode with $\alpha = 0$ is always the most unstable mode. Marginal stability curves for various wavenumber combinations are plotted in Fig. 1.

NONLINEAR TWO-DIMENSIONAL SECONDARY FLOW

Given the results arising from linear stability analysis, and also from DNS and experimental studies [2, 1], we seek a streamwise-independent, steady, nonlinear solution to the governing equations. Nonlinear perturbation equations are solved using the Newton-Raphson algorithm. We find nonlinear solutions which arise from bifurcations from the basic flow which are of a streamwise-vortex character. In Fig. 2 we plot a measure of the amplitude of the nonlinear solution against R for two different wavenumbers. It may be observed that the $\beta = 2.5$ branch joins with the $\beta = 5$ branch at $R \approx 153$. The secondary stability analysis described in the following section finds that the $\beta = 5$ solution loses stability to the $\beta = 2.5$ solution in a streamwise-independent steady, subharmonic bifurcation. Due to the existence of this type of instability, and the fact that lower-order eigenmodes may become unstable, multiple solution branches may exist for a given wavenumber and a rich structure of relationships exists between the two-dimensional nonlinear solutions.

STABILITY OF SECONDARY FLOWS

It is of interest to analyse the stability of the secondary flows described in the previous section, both to determine where these flows may be expected to exist, and also to identify bifurcation points for the tertiary flows. Since the secondary flows are periodic in the spanwise direction Floquet theory applies, and we analyse the stability of a given secondary flow to a linear disturbance with spanwise Floquet parameter, b, and streamwise Floquet parameter, d. Various types of bifurcation have been found which include those which correspond to the steady, spanwise bifurcation described above, and also those which may correspond to the travelling-wave solution described in the introduction. We found that the

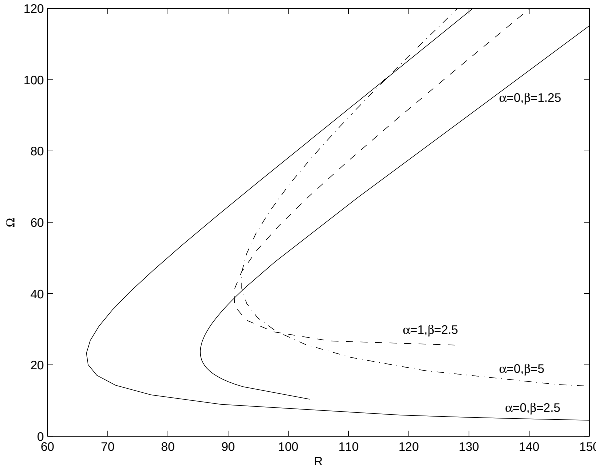

Figure 1. Marginal stability curves corresponding to the most unstable eigenvalue for the values of α and β indicated.

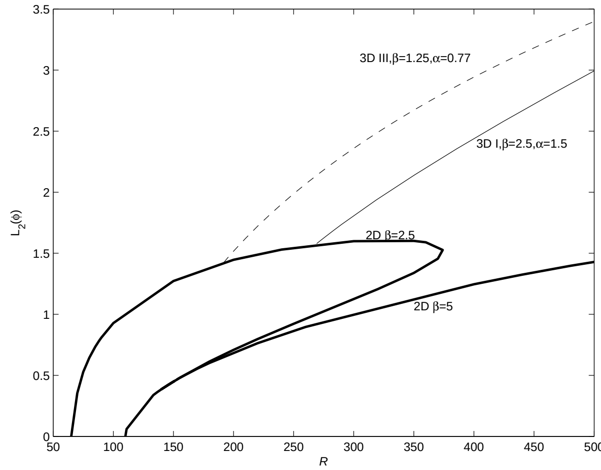

Figure 2. Amplitude of the bifurcating two-dimensional secondary and three-dimensional tertiary flows (I, superharmonic; III, subharmonic) against R when $\Omega = 22.1325$.

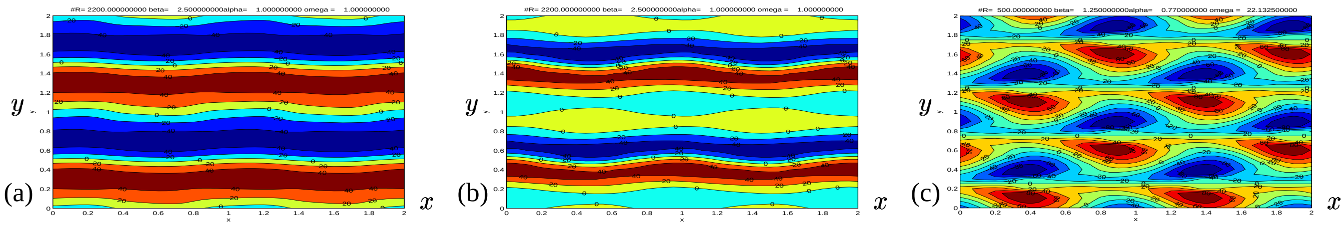

Figure 3. Contours of the streamwise component of vorticity in a streamwise (x) - spanwise(y) slice for tertiary flows (a) superharmonic (I) and (b) superharmonic (II), both for $R = 2200$, $\beta = 2.5$, $\alpha = 1.5$ and $\Omega = 1$, and (c) subharmonic (III) $R = 500$, $\beta = 1.25$, $\alpha = 0.77$ and $\Omega = 22.1325$. The coordinates have been scaled to show two wavelengths in either direction.

fbw bifurcating from the minimal critical point is stable to all disturbances for a fi nite interval beyond the bifurcation point, which is in agreement with the DNS and experimental results [1, 2]. All the fbws appear to fi rst lose stability to an Eckhaus disturbance, i.e. disturbances with $d = 0$. However, the secondary fbw above the critical point remains stable within a closed Eckhaus region. Excellent agreement is found with the experimental study of Afredsson & Persson [1], both for Eckhaus and travelling-wave instabilities.

THREE-DIMENSIONAL TRAVELLING-WAVE TERTIARY FLOWS

We have solved the governing equations for three-dimensional travelling-wave tertiary fbws which arise from a bifurcation from the secondary fbws described above. Three distinct tertiary fbws have been obtained, two superharmonic fbws and one subharmonic fbw. The superharmonic fbws are characterized by staggered vortices lying either side of low-speed streaks in the streamwise velocity component. One of these fbws (I) has one such streak per spanwise wavelength, the other (II) has two. The subharmonic fbw (III) also has vortices lying either side of low-speed streamwise streaks, but these vortices are not staggered. An illustration of these fbws is provided in Fig. 3, while two of the fbws' bifurcations from the secondary fbw are shown in Fig. 2.

CONCLUSIONS

We have analysed the linear stability of rotating Poiseuille fbw and found the steady, streamwise-independent nonlinear bifurcating secondary fbws. The secondary fbw bifurcating from the linear critical point is found to be stable to all disturbances for Reynolds numbers in a fi nite interval above the critical point. A number of possible types of tertiary fbw bifurcation points have been identifi ed. Full nonlinear solutions for three different classes of three-dimensional travelling-wave tertiary fbws are presented. Good agreement is found with previous experimental and DNS-based studies.

References

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