THREE-DIMENSIONAL AIRWAY REOPENING — FINITE-REYNOLDS-NUMBER EFFECTS

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Summary Motivated by the physiological problem of airway reopening, we study the steady propagation of an air finger into a fluid-filled, buckled elastic tube. The resulting three-dimensional, fluid-structure-interaction problem is solved numerically by a fully-coupled, finite-element method. A characteristic two-branch behaviour in the propagation velocity-pressure curve is similar to earlier two-dimensional models [1, 3], and we find that fluid inertia has a significant effect, even at low values of the Reynolds number.

INTRODUCTION

The pulmonary airways are elastic vessels lined with a thin liquid film. Such vessels are susceptible to a fluid-elastic instability that can lead to their collapse and occlusion when the liquid in the film redistributes to form a liquid bridge [4]. The reopening of collapsed airways is believed to occur via the propagation of an air finger into the lungs [1]. The aim of any treatment is to ensure that the propagating air finger clears the liquid blockage and reopens the airways as quickly as possible, but without damaging the lungs. Consequently, the major aim of the theoretical and experimental studies is to determine the propagation speed of the air finger, U, as a function of the applied bubble pressure, p_b^*.

THE MODEL

We model airway reopening by the steady propagation of an inviscid air finger into a buckled, elastic tube containing an incompressible, Newtonian liquid of viscosity μ, density ρ and a constant surface tension, σ^*, at the air-liquid interface. The internal pressure of the air finger is p_b^* and its propagation speed is U. The tube has an undeformed radius R, wall thickness h, Young's modulus E, Poisson's ratio, ν and the cross-sectional area of the tube far ahead of the air finger is A_∞^*. There are four dimensionless parameters governing the system:

$$Ca = \frac{\mu U}{\sigma^*}, \quad Re = \frac{\rho U R}{\mu}, \quad \sigma = \frac{\sigma^*}{RK} \text{ and } A_\infty = \frac{A_\infty^*}{4R^2}, \tag{1}$$

where $K = E(h/R)^3/12(1-\nu^2)$ is the bending modulus of the tube. Ca, the capillary number, is the ratio of viscous to surface-tension forces; Re, the Reynolds number, is the ratio of inertial to viscous forces; σ is the dimensionless surface tension, which represents the ratio of surface-tension forces to the tube's bending stiffness; and A_∞ is the dimensionless cross-sectional area and is a measure of the initial degree of collapse of the tube.

In a moving frame of reference, where the finger tip is fixed at the origin, the fluid motion is governed by the dimensionless, steady Navier–Stokes equations:

$$Re\, \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p + \nabla^2 \boldsymbol{u} \quad \text{and} \quad \boldsymbol{\nabla} \cdot \boldsymbol{u} = 0, \tag{2}$$

where $\boldsymbol{u}$, the fluid velocity, is non-dimensionalised on U; R is the reference length-scale; and the fluid pressure, p, is non-dimensionalised on the viscous scale.

The behaviour of the elastic tube is described by geometrically non-linear Kirchhoff–Love shell theory, in which the deformation of the tube is described only by the deformation of its midplane. The theory include effects due to both stetching and bending of the tube wall, but assumes that the strains remain small. The governing equation follows from the principle of virtual displacements:

$$\iint E^{\alpha\beta\gamma\delta} \left(\gamma_{\alpha\beta} \delta\gamma_{\gamma\delta} + \frac{1}{12}\left(\frac{h}{R}\right)^2 \kappa_{\alpha\beta}\delta\kappa_{\gamma\delta} \right) \mathrm{d}S_0 = \frac{1}{12}\left(\frac{h}{R}\right)^3 \frac{1}{1-\nu^2} \iint \left(\frac{R}{h}\right) \boldsymbol{f} \cdot \delta\boldsymbol{R}\, \mathrm{d}S, \tag{3}$$

where the Einstein summation convention is used, and $\delta\boldsymbol{R}$ denotes the variations of the position vector to the deformed midplane, $\boldsymbol{R}$; $E^{\alpha\beta\gamma\delta}$ is the plane stress stiffness tensor, non-dimensionalised by Young's modulus; $\gamma_{\alpha\beta}$ and $\kappa_{\alpha\beta}$ are tensors that quantify the stretching and bending, respectively, of the midplane, and $\delta\gamma_{\gamma\delta}$ and $\delta\kappa_{\gamma\delta}$ are their variations; S_0 is the surface of the undeformed midplane; finally, $\boldsymbol{f} = \boldsymbol{f}^*/K$ is the traction per unit area of the deformed midplane, S.

The interaction between the fluid and the elastic shell is imposed via two equations. Firstly, the no-slip boundary condition implies that the fluid velocity on the tube wall must be the same as the local wall velocity. Secondly, the fluid exerts a traction on the shell, and the load terms in the solid equations (3) are given by

$$f_i = P^{(\text{ext})} N_i - \sigma Ca \left(p N_i - \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) N_j \right), \tag{4}$$

where $\boldsymbol{N}$ is the (inward) normal to the deformed shell midplane and $P^{(\text{ext})}$ is the external pressure, non-dimensionalised with respect to the bending stiffness, K.

NUMERICAL RESULTS

The coupled system described above was discretised by finite elements and a direct, monolithic solver, based on Newton's method, was used to solve the resulting sets of non-linear algebraic equations.

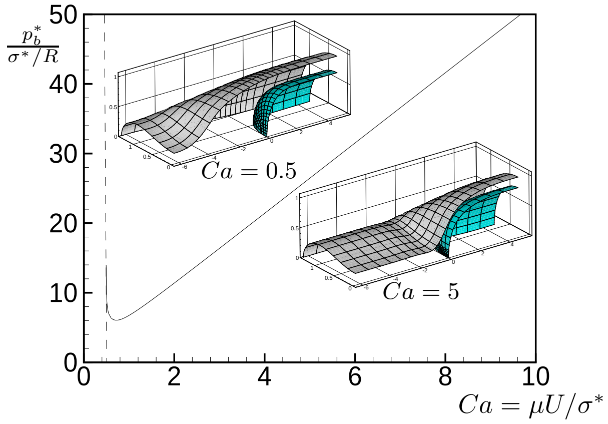

Figure 1. Bubble pressure vs. capillary number in the absence of fluid inertia (zero Reynolds number) for generic system parameters ($\nu = 0.49$, $h/R = 1/20$, $\sigma = 1$, $A_\infty = 0.373$). The dashed line is an asymptotic approximation for the behaviour on the 'pushing' branch. Inset figures illustrate tube and interface shapes on the two branches (adapted from Hazel & Heil 2002).

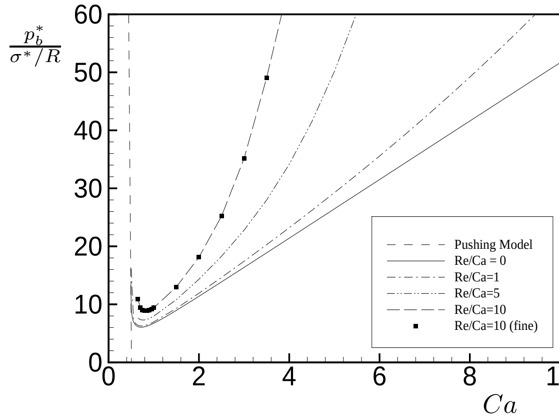

Figure 2. Bubble pressure vs. capillary number for the same parameters as Figure 1, but $Re/Ca = 0, 1, 5, 10$. The markers show the results for $Re/Ca = 10$ on a refined mesh (83,000 dofs), which differ by less than 0.5% from the results at the standard resolution (43,000 dofs).

The behaviour of the system in the absence of fluid inertia, $Re = 0$, was investigated by Hazel & Heil [2], and a typical p_b^*–U curve for the 3D model is shown in figure 1 and is similar to the behaviour observed in previous two-dimensional models [1, 3]. At high speeds, the physically expected behaviour is observed and an increase in bubble pressure causes an increase in propagation speed. In this regime, termed the 'peeling' branch, the tip of the bubble is close to the tube walls and appears to 'peel' them apart, see inset in figure 1. At low speeds, the behaviour changes and a decrease in bubble pressure is required to increase the propagation speed of the air finger. In this regime, termed the 'pushing' branch, a large volume of fluid is 'pushed' ahead of the bubble tip, now located far from the tube walls, see second inset in figure 1.

In any experimental setup, consisting of a given working fluid and a given elastic tube, σ will be constant, as will the ratio $Re/Ca = \rho R\sigma^*/\mu^2$, which depends only upon material parameters and not on the bubble speed, U. We shall therefore assess the effects of fluid inertia on the system by examining curves of bubble pressure versus propagation speed for fixed values of the ratio Re/Ca, rather than for fixed values of Re. Figure 2 shows such curves for $Re/Ca = 0, 1, 5$ and 10, and as in Heil's [3] two-dimensional model, even relatively weak fluid inertia has a very strong effect on the system's behaviour. In particular, compared to the case of zero Reynolds number, much larger bubble pressures are required to drive the air finger at a given capillary number. Inertial effects become more pronounced as Ca increases, owing to the corresponding increase in $Re = \text{const} \times Ca$. Conversely, at small values of Ca, and hence Re, inertial effects become negligible and Hazel & Heil's [2] 'pushing' model provides a good approximation for the system's behaviour for all values of Re/Ca.

References

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