STABILITY OF SHEAR-FLEXIBLE FRAMES

Esben Byskov, Ronni Dam, Thomas Frost and Lars Hulsbæk
Department of Building Technology and Structural Engineering, Sohngaardsholmsvej 57, Aalborg University, DK-9000 Aalborg, Denmark

Summary

INTRODUCTION

Most studies of postbuckling and imperfection sensitivity of frame structures have assumed shear-inflexibility. Here, we present theoretical, numerical and experimental results for a shear flexible realization of a particular frame, the so-called "Roorda Frame," see Fig. 1. The frame consists of two sandwich beams of length 916 mm

(a) The Roorda Frame.

(b) Joint and loading device.

Figur 1. Shear-flexible Roorda Frame with loading device.

connected through a very stiff joint, see Figs. 1(a) and 1(b), which also show the loading device. The core is made of a foam (Divinycell H130) and the face plate material is a high-strength aluminum (2024-T3). The depth of the members is 43.2 mm and the thickness of the face plates is 1.6 mm. All deformations occur only in the plane of the frame.

By moving the loading device, different levels of load imperfection can be introduced.

When dealing with sandwich beams and frames it is very important noticing that the idea of a stiff joint must be reconsidered simply because it is the rotation of the *cross-section*, not the rotation of *beam axis*, which is continuous over the joint. This means that the horizontal member, the beam, does not support the vertical one, the column, as well as in the Bernoulli-Euler case.

POSTBUCKLING AND IMPERFECTION SENSITIVITY

Let u be the axial and w the transverse displacement, respectively, and let ω denote the rotation of the beam cross-section. Then, the generalized strains of a moderately nonlinear Timoshenko beam theory are:

$$\text{Axial strain}: \quad \varepsilon = u' + \tfrac{1}{2}(w')^2\,, \quad \text{Shear strain}: \quad \gamma = w' - \omega\,, \quad \text{Curvature strain}: \quad \kappa = \omega' \tag{1}$$

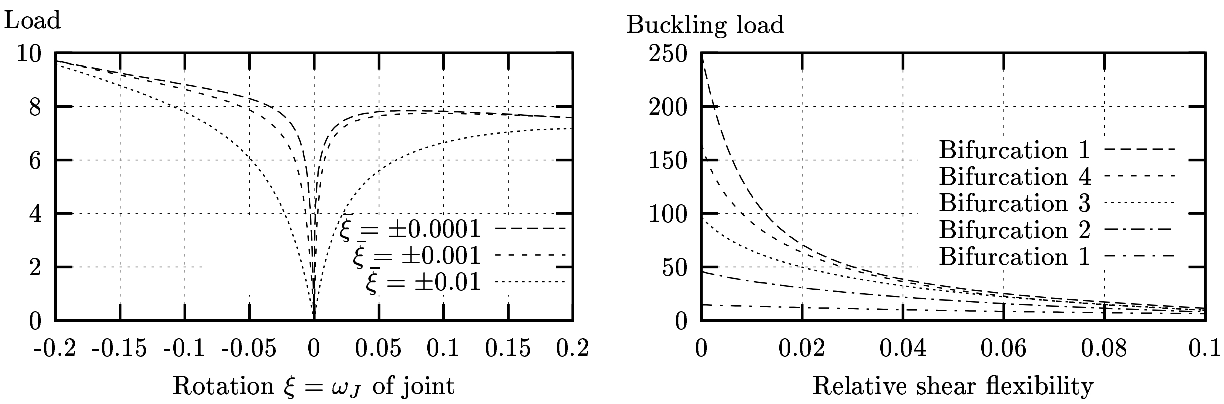
Thus, we may apply the asymptotic theory of postbuckling and imperfection theory first developed by Koiter, see e.g. [1] or [2]. Koiter has solved the problem of postbuckling and imperfection sensitivity of the same kind of frame using Bernoulli-Euler beam theory and assuming inextensibility, see [3]. The latter assumption is also approximately valid for frames such as the present one. According to Koiter [1], for an unsymmetric structure the the load-carrying capacity λ_S of a geometrically imperfect realization of the frame may be determined as the maximum of λ on the relation:

$$\left(1 - \frac{\lambda}{\lambda_C}\right)\xi + a\xi^2 = \frac{\lambda}{\lambda_C}\bar{\xi} \tag{2}$$

where λ denotes a scalar load parameter, λ_C the classic critical load of the perfect structure, ξ the amplitude of the buckling mode, a the (first-order) postbuckling constant, and $\bar{\xi}$ designates the amplitude of the geometric imperfection, which is assumed to be affine with the buckling mode. Once the prebuckling and buckling fields have been determined, the value of a is easily computed. For the original Roorda Frame the buckling mode amplitude is usually chosen as the rotation ω_J of the joint. This is also done here. Below, finite element results, which agree asymptotically with the analytic solutions, but cover a larger range of amplitudes, are compared with experimental results.

FINITE ELEMENT STUDY AND EXPERIMENTAL RESULTS

Unless special elements are utilized the phenomenon of "locking" occurs because the beam theory accounts for shear flexibility. Since the applied beam theory accounts for shear flexibility, usual beam finite elements display "locking" with the consequence that the results become unreliable. Application of special elements handles this problem. Equilibrium paths for various levels of geometric imperfections in the shape of the first buckling mode are shown in Fig. 2(a).



(a) Equilibrium paths of geometrically imperfect sandwich frames. Rotation of joint in radian, positive counter-clockwise. Imperfection levels are given by $\bar{\xi}$.

(b) Variation of bifurcation load with increased shear flexibility.

Figur 2. Finite element results

If the frame consists of a shear-inflexible material the buckling loads are well separated, see Fig. 2(b). The figure also shows that this separation decreases for increasing (relative) shear flexibility. This effect must be accounted for in the numerical procedures in order to avoid inaccuracies.

Comparison between experimental and finite element results shows fair agreement. An important reason for the discrepancies stems from the applied beam theory in that the assumption of cross-sections remaining plane after deformation is too crude to model sandwich beams correctly. In addition to, and associated with this, determination of the beam cross-sectional properties, i.e. the bending and shear stiffness, is not a straightforward issue. The foam, although of very high quality, is neither homogeneous nor isotropic on the global scale.

CONCLUDING REMARKS

Determination, both experimentally and numerically, of postbuckling behavior and imperfection sensitivity of frames of sandwich materials presents serious difficulties in comparison with the equivalent problem for frames made of materials that display little shear-flexibility. Modeling of the cross-sectional properties must be given special attention. Special finite elements must be used in order to avoid shear locking.

References

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