A NON-ASSOCIATIVE ANISOTROPIC DAMAGE MODEL FOR BRITTLE MATERIALS

Luigi Gambarotta, Ilaria Monetto
DISEG, Department of Structural and Geotechnical Engineering, University of Genoa, Italy

Summary A friction-damage coupled model is derived from a micromechanically based description of the response of elastic matrices embedding statistically uniform distributions of crack-like defects. Symplifying assumptions on the mathematical representation of internal variables, governing the microscopic mechanisms responsible for the global inelastic response, permit a relatively simple formulation capable of describing the load-induced anisotropy of damage.

BACKGROUND

Inelastic behavior of materials responding in a predominantly brittle mode is the result of nucleation and propagation of crack-like microdefects, commonly referred to as damage, that may be accompanied by residual deformation due to possible frictional resistance to sliding of the internal crack surfaces. In the framework of thermodynamics with internal variables, convenient for dealing with the constitutive modeling of such dissipative materials, most literature has focused on modeling phenomenologically and micromechanically the progressive degradation of mechanical properties due to damage evolution rather than on the sliding-damage interaction [1]. Restricting the analysis to plane loading conditions, models for brittle solids under compression based on the sliding crack mechanism are a first step. As dual approach to the rigorous micromechanical model presented by Nemat-Nasser and Obata [2], Basista and Gross [3] in particular re-examined the sliding crack mechanism within the framework of thermodynamics with internal variables but deriving the potential functions from micromechanics.

CONSTITUTIVE MODEL

In order to analyze the different response of brittle and quasi-brittle materials in tension and compression, a non-associative anisotropic damage model incorporating the damage – frictional sliding interaction is derived in this paper. Brittle materials are modelled as elastic isotropic matrices embedding statistically uniform distributions of crack-like defects. The mean strain $\mathbf{E}$ may then be split into the elastic contribution due to the elastic matrix (having elastic compliance fourth-order tensor $\mathbb{K}$) and the inelastic contributions $\mathbf{E}_n$ and $\mathbf{E}_t$ due to microcrack opening and sliding, respectively:

$$\mathbf{E}=\mathbb{K}\mathbf{T}+\mathbf{E}_n+\mathbf{E}_t\,, \tag{1}$$

where $\mathbf{T}$ is the mean stress. Under the hypotheses of non-interacting and self-similarly propagating flat cracks, the analysis builds on that of Gambarotta and Lagomarsino [4] based on the concept of damage planes. The ensemble of microcracks is thought of as consisting of sets of identical equi-oriented flat cracks. The state of each set, identified by the orientation of the related crack plane, say $\mathbf{n}\in\Omega$ (the unit hemi-sphere in $\Re^3$ being denoted by Ω), can be defined by the characteristic size of the cracks, say $a(\mathbf{n})$, and by the vector of tangential contact tractions, say $\mathbf{f}(\mathbf{n})$; whereas, under the assumption of frictional sliding with no extension, normal contact tractions are given by compressive stresses. On the other hand, the introduction of such orientation fields as internal variables makes the model too cumbersome to have any practical appeal in a generic loading program.
The strategy applied in this paper for the reduction of the computational effort uses approximate representations of the orientation fields mentioned above in terms of appropriate series expansion of generalized spherical harmonics [5, 6], so allowing the forms:

$$a(\mathbf{n})\approx\mathbf{n}\bullet\mathbf{A}\mathbf{n}\,, \tag{2}$$

as previously assumed in [7], and, by analogy with the shearing stresses:

$$\mathbf{f}(\mathbf{n})\approx(\mathbf{I}-\mathbf{n}\otimes\mathbf{n})\mathbf{F}\mathbf{n}=(\mathbf{I}-\mathbf{n}\otimes\mathbf{n})\mathbf{F}'\mathbf{n}\,. \tag{3}$$

A more convenient description for damage and contact tractions in terms of two symmetric second-order tensors $\mathbf{A}$ and $\mathbf{F}$ (namely, the deviatoric component $\mathbf{F}'$ with tr$\mathbf{F}'$=0 as the only significant component of $\mathbf{F}$) as new overall internal variables is then obtained. Note that at the undamaged state the damage tensor $\mathbf{A}=\mathbf{0}$, whereas during the damage process it is positive definite due to constraints provided by the irreversibility condition of damage. As for the internal friction tensor, $\mathbf{F}'=\mathbf{0}$ iff all the crack planes are subjected to tension. Using (2) and (3), the inelastic contributions to the mean strain shown in (1) can be specialized and the Gibbs free-energy function so derived from micromechanics permits one to associate $\mathbf{F}'$ and $\mathbf{A}$ with their thermodynamically conjugate variables: respectively, the sliding strain tensor $\mathbf{E}_t^-$ related to the closed cracks on compressive damage planes ($\mathbf{n}\in\Omega^-$, having defined $\Omega^-=\Omega-\Omega^+$ together with $\Omega^+=\{\mathbf{n}\in\Omega\,|\,\mathbf{n}\bullet\mathbf{T}\mathbf{n}\geq 0\}$) and the damage energy release rate tensor $\mathbf{Y}$. The latter in particular is a symmetric, positive definite second-order tensor.
The constitutive equation is finally complemented by limit conditions, governing potential activation of dissipation mechanisms, and equations of evolution for the associated internal variables, describing the way irreversible processes evolve. According to the simplified formulation based on overall (tensor-valued) variables, overall frictional sliding and crack growth criteria have been assumed.

RESULTS AND CONCLUSIONS

Because of nonlinearity and stress-path-dependence of the constitutive response, the model should be necessarily formulated in incremental form to predict the constitutive response to arbitrary stress applied to a brittle material at an arbitrary current state and requires, in general, numerical analyses to perform. Proportional loading, under which no crack changes its status is an exception and explicit solutions are possible as well. With reference to initiallly undamaged materials, biaxial and triaxial failure envelopes, together with some characteristic stress-strain curves, have been obtained and used for both identification and validation of the model.

As an example, figures 1, 2 and 3 show the biaxial limit strength domain and some related stress–strain curves, computed for some values of model parameters. Note that: (i) nonlinearity of the overall responses reflects damage evolution, which in the specific case of compression tests is accompanied by frictional sliding; (ii) if unloading is initiated, since damage evolution is locked, a linearly elastic tensile response is observed, whereas in compression the bilinear unloading branch reflects the occurence of backsliding. These results in general confirm the previous Brencich and Gambarotta [8] simulations but the use here of a second-order tensor rather than a single scalar variable permits a more convenient description for the load-induced anisotropy (namely, orthotropy) of damage.

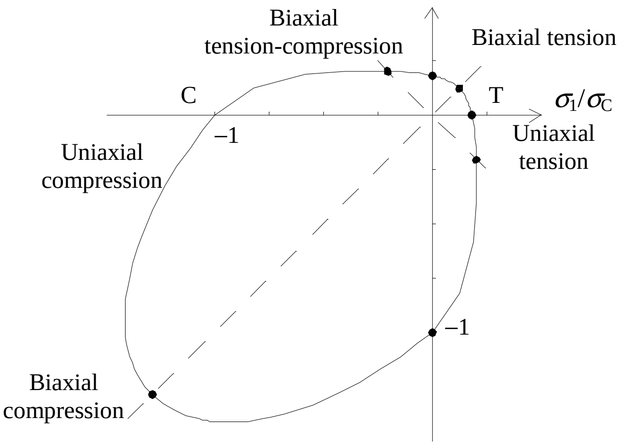

Figure 1. Biaxial limit domain.

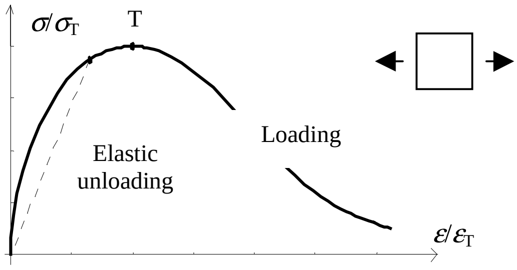

Figure 2. Model response to uniaxial tension.

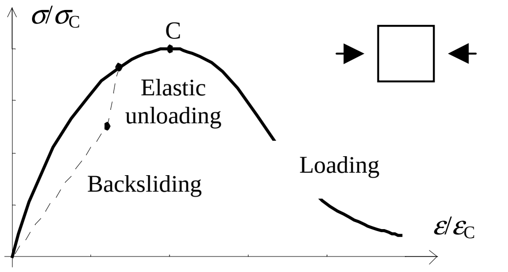

Figure 3. Model response to uniaxial compression.

It is straightforward that, because of the assumption of statistically uniformly distributed damage as an ensemble of non-interacting microcracks, the model is inherently incapable of predicting the onset of macroscopic failure occurring as damage localization into bands and coalescence of microdefects into a macrocrack. Practical application of the model is then restricted to the hardening phase of the mechanical response of brittle and quasi-brittle materials. Furthermore, due to the self-similar crack growth, the characteristic feature of dilatancy cannot be captured. On the other hand, with all of its limitations, in this model effects of anisotropic damage and sliding for prescribed (even complex) loading are combined in a relatively simple formulation which takes the physics of the problem into account.

References

[1] Krajcinovic D.: Damage mechanics: accomplishments, trends and needs. *Int J. Solids Struc* **37**: 267-277, 2000.
[2] Nemat-Nasser S., Obata M.: A microcrack model of dilatancy in brittle materials. *J. Applied Mech* **55**: 24-35, 1988.
[3] Basista M., Gross D.: The sliding crack model of brittle deformation:an internal variable approach. *Int J. Solids Struc* **35**: 487-509, 1998.
[4] Gambarotta L., Lagomarsino S.: A microcrack damage model for brittle materials. *Int J. Solids Struct* **30**: 177-198, 1993.
[5] Kanatani K.I.: Distribution of directional data and fabric tensors. *Int J. Engng Sci* **22**: 149-164, 1984.
[6] He Q.C., Curnier A.: A more fundamental approach to damaged elastic stress-strain relations. *Int J. Solids Struct* **32**: 1433-1457, 1995.
[7] Gambarotta L.: Friction-damage coupled model for brittle materials. *Eng Fracture Mech* **71**: 829-836, 2004.
[8] Brencich A., Gambarotta L.: Isotropic damage model with different tensile-compressive response for brittle materials. *Int J. Solids Struct* **38**: 5865-5892, 2001.