OPTIMAL ENERGY GROWTH AND OPTIMAL CONTROL OF THE SWEPT ATTACHMENT-LINE BOUNDARY LAYER

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Summary An adjoint formulation is used to find the optimal perturbation for the swept attachment-line boundary layer in the context of the Görtler-Hämmerlin formulation. Two-dimensional mechanisms in a plane normal to the plate are shown to generate transient energy growth. Optimal control based on wall-normal blowing and suction is demonstrated to be very efficient and to strongly decrease the energy amplification of the optimal perturbation both in the linearly stable and linearly unstable régimes.

INTRODUCTION

The growth of disturbances in the vicinity of the leading-edge of swept wings is known to trigger transition to turbulence over the entire wing surface. It is therefore crucial to develop a control methodology capable of quenching developing disturbances. The three-dimensional swept Hiemenz flow boundary layer along the attachment line (figure 1) is known to become linearly unstable above a Reynolds number of 583 [1]. Furthermore strong transient growth possibly leading to by-pass transition has recently been demonstrated [2] [3].

The objective of the present study is to implement a Lagrangian-based variational optimization scheme [4] in order to first determine optimal perturbations living in swept Hiemenz flow under the Görtler-Hämmerlin assumption. In a second step, the same scheme is adapted to find the optimal control sequence of wall blowing and suction capable of inhibiting the growth of disturbances.

SWEPT HIEMENZ FLOW AND GÖRTLER-HÄMMERLIN FORMULATION

Swept Hiemenz flow is an exact solution of the incompressible Navier-Stokes equations that describes the steady swept attachment-line boundary layer as sketched in figure 1. The velocity components along the $(x,\ y,\ z)$ axes being denoted $(U,\ V,\ W)$, swept Hiemenz flow is obtained by assuming

$$U = xRe^{-1}F'(y)\ , \quad V = -Re^{-1}F(y)\ , \quad W = W(y)\ ,$$

where $\psi = -xRe^{-1}F(y)$ denotes the stream function and Re the Reynolds number based on the freestream spanwise velocity.

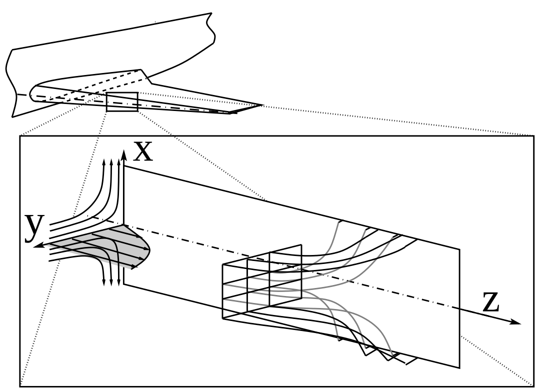


Figure 1. Sketch of swept attachment-line boundary layer.

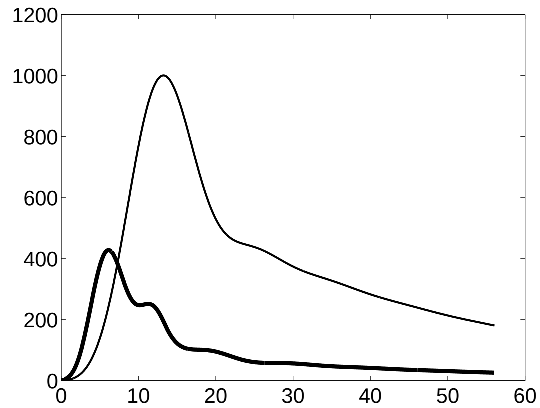


Figure 2. Energy of the optimal perturbation versus time when no control (thin line) or optimal control (thick line) is applied. Parameter settings : $Re = 2000$, $k = 0.1$, $T_p = 14$, $T_c = 14$, $l = 0.5$. Linearly stable base flow.

Under the Görtler-Hämmerlin assumption, small perturbations $(\check{u}(y,z,t),\ \check{v}(y,z,t),\ \check{w}(y,z,t))$ are superimposed on the base flow so as to model the perturbed flow $(u,\ v,\ w) = (U + x\check{u}, V + \check{v}, W + \check{w})$. At leading order, the Navier Stokes equations reduce to a linear system involving $\check{u}$ and $\check{v}$. Fourier decomposition in the spanwise direction leads to a linear system of the form

$$\left(A_{(Re,k)}\partial_t + B_{(Re,k)}\right)\begin{bmatrix}\hat{u}_k & \hat{v}_k\end{bmatrix}^t = 0 \tag{1}$$

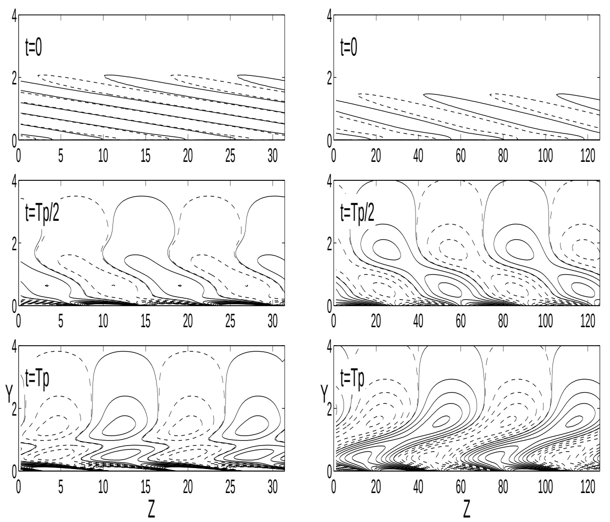
governing the evolution of the Fourier coefficients $(\hat{u}_k,\ \hat{v}_k)$ for a given spanwise wavenumber k. The symbols $A_{(Re,k)}$ and $B_{(Re,k)}$ denote linear differential operators in y only.

OPTIMAL PERTURBATIONS AND OPTIMAL CONTROL

When no wall blowing is applied, the *optimal perturbation* at a given time T_p denotes the initial condition that leads to the maximum kinetic energy at T_p ; the *optimal control* at a given time T_c is the time sequence of wall-normal blowing and suction that leads to the minimum energy at time T_c for a given initial condition. If $E(t)$ denotes the energy of the perturbations at time t, the optimal perturbation and the optimal control correspond to extrema of the *cost function* $\mathcal{I} = E(T_{opti})/E(0) + (l^2/2)||\hat{v}(y=0)||^2$, where T_{opti} is either T_p or T_c and l a control parameter weighing the cost of control. A Lagrangian-based variational method, that requires to solve the *adjoint problem* of system (1) is implemented to find these extrema, at fixed $(Re,\ k)$. An example of the transient energy growth of an optimal perturbation is given in figure 2 (thin line) in the linearly stable régime. Under optimal control (figure 2, thick line), the energy amplification at time $T_c = 14$ is seen to be decreased from 1000 to 155.

UNDERLYING PHYSICAL MECHANISMS

At any $(Re,\ k)$, the optimal perturbation (figure 3) consists of vortices aligned in the chordwise direction x and tilted against the sweep W. Depending on the spanwise wavenumber k, two distinct behaviours have been observed, that both lead to transient energy growth : when $k > 0.3$ (figure 3a), the vortices are tilted by the basic shear as in the well-known 'Orr mechanism'. When $k < 0.3$ (figure 3b), the vortices split and re-arrange with their nearest neighbour, in addition to the previous tilting. A parameter study reveals that this mechanism is responsible for the strongest energy growth.



a b

Figure 3. Vorticity contours of the optimal perturbation in the (z, y) plane at time $t = 0,\ T_p/2,\ T_p$. Wall is at the bottom and sweep is from left to right. Parameter settings : $Re = 2000$, (a) $k = 0.4$ and $T_p = 24$, (b) $k = 0.1$ and $T_p = 14$.

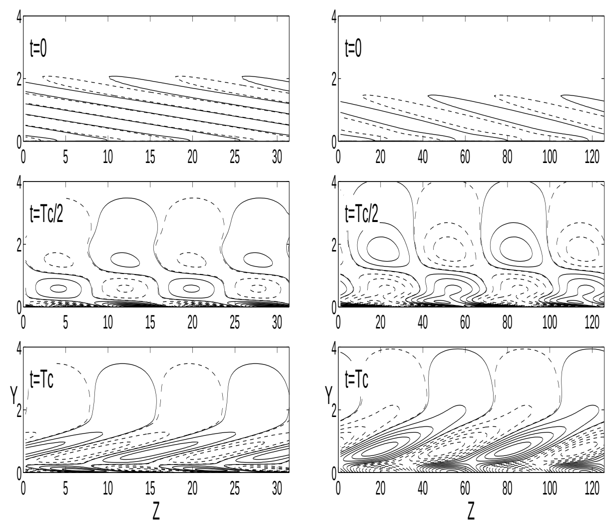


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Figure 4. Vorticity contours in the (z, y) plane of the optimal perturbation when control is applied, at $t = 0,\ T_c/2,\ T_c$. Wall is at the bottom and sweep is from left to right. Parameter settings : $l = 0.5$, $Re = 2000$, (a) $k = 0.4$ and $T_c = 24$, (b) $k = 0.1$ and $T_c = 14$.

According to figure 4, optimal control affects the flow in such a way that the vortices evolve faster than they would without wall blowing, so as to lead to a minimum energy at time T_c. This leaves no time for the perturbations to extract energy from the mean flow, and inhibits energy amplification. Such an optimal control is shown to be very similar to opposition control in the linearly unstable régime.

The present study has demonstrated that closed loop active control based on wall-normal blowing and suction effectively damps the energy amplification of perturbations, thereby delaying transition (without control, these disturbances would experience dramatic energy growth).

References

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