KINETICS OF WEAKLY FRICTIONAL GRANULAR GASES

Isaac Goldhirsch*, S. Henri Noskowicz*, O. Bar-Lev**
Department of Fluid Mechanics and Heat Transfer, Faculty of Engineering, Tel-Aviv University, Ramat-Aviv, Tel-Aviv, 69978, Israel
**School of Mathematics, Tel-Aviv University, Ramat-Aviv, Tel-Aviv, 69978, Israel.*

<u>Summary</u> Although all macroscopic grains are frictional, most of the theoretical studies of granular gases have focused on models of frictionless spheres. The success of these models in explaining many experimental results raises the question of the role of frictional restitution in the dynamics of granular gases. The weak frictional limit is explored here. Using the pertinent Boltzmann equation we derive hydrodynamic equations of motion (as it turns out, one needs to extend the list of hydrodynamic fields for this purpose) and study some of their consequences. One of these consequences is that the homogeneous cooling state is highly non-Gaussian. Another consequence is the long memory associated with the rotational degrees of freedom, when weakly coupled to translation.

EXTENDED DESCRIPTION

In spite of the omnipresence and undeniable importance of friction in granular matter (see e.g., [1]), most models of granular gases consist of frictionless particles, mostly spherical in shape. Although frictional particles have been considered in simulations of granular gases, cf. e.g., [2] and references therein, very few theoretical studies of frictional granular gases can be found in the literature, cf. e.g., [3, 4, 5, 6]. Perhaps the fact that frictionless models have been rather successful in explaining many of the observed phenomena in granular gases [7], or the difficulties one encounters in the theoretical study of frictional granular gases are responsible for this state of affairs, and perhaps there are other reasons. In any case, we believe it is important to study the full kinetics and hydrodynamics of frictional granular gases.

In previous kinetic theoretical based studies of the of granular hydrodynamics [3, 5] it is assumed that the basic distribution function is Maxwellian in both the velocity and angular velocity (and usually different rotational and translational temperatures are allowed for), and corrections due to gradients are identified (on the basis of symmetry). The assumed distribution function is substituted in the Enskog equations, resulting in a closure for the constitutive relations. The above Maxwellian distribution corresponds (when both the rotational and translation temperature are taken to be equal) to the limit of rough molecules, in which there is equipartition between the rotational and translational kinetic energies. Our goal here is to study granular gases near a different limit, i.e., the smooth limit (as frictionless grains comprise the model most studied in the literature), and develop a systematic approach to the problem, i.e., a perturbative scheme

The model considered here is a monodisperse systems of spherical grains. In addition to the velocities of the particles we define a 'spin variable', $\mathbf{s}_i$, whose dimension is that of velocity, by $\mathbf{s}_i \equiv \frac{d}{2}\omega_i$, where d is the diameter of a sphere and ω_i is its angular velocity. The binary collisions of these spheres are defined similarly to refs. [5, 8]. The single particle distribution function, f, depends on space ($\mathbf{r}$), time, t, the velocity, $\mathbf{v}$, and spin, $\mathbf{s}$, degrees of freedom, and it satisfies and appropriate Boltzmann equation. The latter can be studied by employing the Champman-Enskog expansion in order to obtain hydrodynamic equations of motion. The basic premise of this expansion is that the dependence of the single particle distribution function on space and time can be replaced by a dependence on the 'slow' fields. In other words, when the temporal resolution exceeds a few mean free times, and the spatial resolution exceeds a few mean free paths, the (local) dynamics of the particles is only constrained by the slowly changing fields. Therefore, the slow (or hydrodynamic) fields should be those that correspond to conserved (or nearly conserved) entities. In the case at hand, the momentum and the number of particles (or the total mass) are conserved, hence the momentum density and the particle number (or mass) density are hydrodynamic fields. The granular temperature corresponding to the translational degrees of freedom is not conserved, but it is nearly conserved in the near-elastic, near-frictionless case, and is therefore a hydrodynamic field. In the same limit the spin degrees of freedom are decoupled from the translational degrees of freedom and the number density corresponding to each value of the spin, $\mathbf{s}$, is conserved as well, hence the infinite set of spin dependent number densities should be taken as hydrodynamic fields. In the rough limit rotation is strongly coupled to translation and the total (translational plus rotational) energy is conserved in each collision. Therefore, in the latter case one only needs the total energy, momentum and mass densities as hydrodynamic fields. However, no harm is done by exaggerating the number of hydrodynamic fields, hence even in this case one can use the above set of spin dependent densities (which can be used to calculate the rotational temperature as the average of the square of the of the fluctuating spin), and in this case one can retain the translational granular temperature as well. All in all, the general set of hydrodynamic fields that is appropriate for all cases is that corresponding to the smooth elastic limit.

There are several ways to design a perturbative expansion for the distribution function in the present case; each of the expansions uses a different zeroth order limit, around which it is defined. It is preferred that this limit is an exact solution of the Boltzmann equation. In the realm of smooth frictionless granular systems there have been two such expansions. One was [9] the equilibrium state corresponding to the limit of zero inelasticity ($e = 1$) and no-gradients (i.e., the Knudsen number, K, was taken to be zero) and, the other [10] employed the (local) homogeneous cooling state (with $K = 0$) as a zeroth order. These two approaches agree with each other in the common domain of validity (near elastic collisions) and both can be used as zeroth order solutions in the presence of weak friction. Another limit at which a solution of the Boltzmann equation can be identified is that of rough molecules, in which case there is a zeroth order solution in which the

rotational and translational temperature are equal to each other. The latter has been employed to describe molecules, cf. [11]. One of the important differences between the smooth particle limit and the rough limit is that in the former the spin degrees of freedom are decoupled from the translational degrees of freedom to zeroth order in the interaction, and serve as independent variables, whereas in the latter case the spin degrees of freedom are strongly coupled to the translational degrees of freedom (the conserved energy in collisions is the sum of the translational and rotational energies) and thus the spin degrees of freedom are not independent. For this reason, and since the smooth limit is the most studied case, we chose to focus on the smooth equilibrium limit. This choice still leaves several possibilities. Below, we choose to study the case of finite Coulomb friction and near frictionless coefficient of tangential restitution. In this case, the small parameters are: the degree of normal inelasticity [9], the degree of tangential inelasticity, and the Knudsen number. Notice that the studied systems can be regarded as a mixture of particles characterized by their respective spins.

The results of the application of the Chapman-Enskog expansion yield constitutive relations, which we have developed to second order in the small parameters. These will be presented elsewhere. Here we specialize to a simple case, the near frictionless homogeneous cooling (i.e., unforced) state. In this case the hydrodynamics equations reduce to:

$$\begin{aligned}
\frac{dT}{d\tau} &= -\alpha T + \beta T_{\text{rot}} + 2\beta S^2 \\
\frac{dT_{\text{rot}}}{d\tau} &= \frac{3}{\kappa^2}\beta T - \left(\delta - \frac{1}{4}\mu - \frac{3}{2\kappa^2}\beta\right) T_{\text{rot}} + \mu S^2 \\
\frac{dS^2}{d\tau} &= -\delta S^2
\end{aligned} \tag{1}$$

where, T is the granular temperature, T_{rot} is the rotational granular temperature, $\mathbf{S}$ is the average spin field, and τ measures time in units of accumulated collisions. . Eq. (1) is linear in T, T_{rot} and S^2. The three eigenvalues of this system are negative. The long-time decay rate of T and T_{rot} follows Haff's law, and: $\lim_{t\to\infty} \frac{T_{\text{rot}}}{T} = r$, where r the value of the number r depends on the parameters in Eq.(1). Also: $\lim_{t\to\infty} \frac{S^2}{T} = 0$. Since S^2 decays to zero faster than T and T_{rot} it is justified to consider the asymptotic time dynamics for the case $\mathbf{S} = 0$. To simplify matters even further, we shall assume that the spin distribution, $n(\mathbf{s}, t)$, is isotropic, i.e., $n(\mathbf{s}, t)$ depends on the scalar s (and time) alone. It is convenient to define a spin density, $N(s,t)$, by $N(s,t) \equiv 4\pi s^2 n(s,t)$. Clearly: $\int N(s,t)ds = n$, and $\frac{1}{n}\int s^2 N(s,t)ds = T_{\text{rot}}(t)$. One can show that N satisfies a spin diffusion equation: $\frac{\partial N}{\partial \tau} = \frac{\partial}{\partial s}\left[A_1 sN + (2T + T_{\text{rot}}) A_2\left(\frac{\partial N}{\partial s} - \frac{2N}{s}\right) + A_3\left(s^2\frac{\partial N}{\partial s} - 2sN\right)\right]$. In the large time limit, this equation possesses a scaling solution of the form $N(s,\tau) = \frac{n}{\sqrt{T}}F(\zeta)$ where $\zeta = \frac{s}{\sqrt{T}}$, with T_{rot}/T replaced by its asymptotic value r. In this limit the above equation reduces to $F_{\zeta\zeta} + \frac{F_\zeta}{\zeta}C + \frac{F}{\zeta^2}(C-2) = 0$, where C is a fixed parameter. It is easy to check that the normalizable solution of the last equation satisfies $F(\zeta) \sim \zeta^{2-C}$ i.e. the distribution decays as a power law: $n(s) \sim \frac{n}{4\pi T^{3/2}}\left(\frac{s}{\sqrt{T}}\right)^{-C}$. For instance for a degree of inelasticity, $\epsilon = 0.1$ and a similar degree of frictional inelasticity, $F(\zeta) \sim \zeta^{-5.935}$. Therefore, contrary to assumptions in most theories of frictional granular matter, the spin distribution function in the considered case is very different from Maxwellian. As the hydrodynamics was developed on the basis of the Boltzmann equation, (dilute case), the stress tensor (being the average of a symmetric entity) is symmetric, i.e., one does not obtain a micropolar theory. Finite density corrections will be considered elsewhere. Many open questions still remain, among them the effects of friction on shear and vibrated flows, the matching of the expansion presented above with results obtained from an expansion around the rough limit, and, in general, the effects of friction in many types of granular flows.

ACKNOWLEDGMENTS

.

References

[1] S. J. Moon, J. B. Swift, and H. L. Swinney. Role of friction in pattern formation in oscillated granular layers. *cond-mat/0308541*, 2003.
[2] R. Cafiero, S. Luding, and H. J. Herrmann. Rotationally driven gas of inelastic rough spheres. *Europhys. Lett.*, 60(6):854–860, 2002.
[3] C. K. K. Lun. Kinetic theory for granular flow of dense, slightly inelastic, slightly rough spheres. *J. Fluid Mech.*, 233:539–559, 1991.
[4] M. Huthmann and A. Zippelius. Dynamics of inelastically colliding rough spheres: Relaxation of translational and rotational energy. *Phys. Rev. E*, 56(6):6275–6278, 1998.
[5] J. T. Jenkins and C. Zhang. Kinetic theory for identical, frictional, nearly elastic spheres. *Physics of Fluids*, 14(3):1228–1235, 2002.
[6] H. Hayakawa N. Mitarai and H. Nakanishi. Collisional granular flow as a micropolar fluid. *Phys. Rev. Lett.*, 88(17):174301–1–4, 2002.
[7] I. Goldhirsch. Rapid granular flows. *Annual Reviews of Fluid Mechanics*, 35:267–293, 2003.
[8] O. Herbst, M. Huthmann, and A. Zippelius. Dynamics of inelastically colliding spheres with Coulomb friction: relaxation of translational and rotational energy. *Granular Matter*, 2(4):211–219, 2000.
[9] N. Sela and I. Goldhirsch. Hydrodynamic equations for rapid flows of smooth inelastic spheres to Burnett order. *J. Fluid Mech.*, 361:41–74, 1998.
[10] J. J. Brey, J. W. Dufty, C. S. Kim, and A. Santos. Hydrodynamics for granular flow at low density. *Phys. Rev. E*, 58(4):4638–4653, 1998.
[11] S. Chapman and T. G. Cowling. *The mathematical theory of nonuniform gases*. Cambridge University Press, London, 1960.