XXI ICTAM, 15–21 August 2004, Warsaw, Poland

MODELING OF PERIODIC LOAD EFFECTS IN BONE TISSUE ADAPTATION

Tomasz Lekszycki
Institute of Fundamental Technological Research, Polish Academy of Sciences
Swietokrzyska 21, PL 00-049 Warszawa, Poland

Summary Bone functional remodeling is influenced by time characteristics of mechanical loading. Periodic loads usually have more effect on the adaptation and associated evolution of tissue structure than a static one. In the present work, an approach, based on the hypothesis of optimal response of bone, is proposed. It enables derivation for a viscoelastic material a family of adaptation formulas. In this derivation, an assumption is made that the bone is loaded harmonically, and that the frequency of oscillations is so small that the inertia effects are negligible. It follows that the stimulus is frequency dependent and that functional relation between stimulus and load frequency depends on the constitutive material model applied in the formulation. In some cases, the stimulus grows to some maximal value and next drops to zero with steering to infinity frequency of load oscillations. The results of sample computer calculations are included. The proposed approach might offer an attractive tool in investigations of frequency effect on bone remodeling but more research and experimental confirmation are necessary.

INTRODUCTION

It has been observed that bone functional remodeling is influenced by time characteristics of mechanical loading. Variable in time loads usually have more effect on the adaptation and evolution of tissue structure then the static one, see e.g. [1]. However this problem needs more theoretical and experimental investigations, see [2]. There exist proposals for adaptation models based on information concerning micro-damage accumulation, where the number of load cycles is included in the formulation, see [3]. In the present work we propose another approach, based on the hypothesis of optimal response of bone introduced in [4] and [5] and developed till now in context of elastic characteristics of bones. It also enables derivation of a family of adaptation laws for viscoelastic materials under an assumption that bone is loaded harmonically, and that the frequency of oscillations is so small that inertia effects can be neglected. It follows from this derivation that stimulus is frequency-dependent. Thus the rate of changes of tissue micro structure depends not only of the amplitude of loads, but also on their frequencies.

DERIVATION OF ADAPTATION RELATIONS

Assumptions

1) Material has viscoelastic characteristics, slowly evolving in time due to bone adaptation. 2) The mechanical load is varying harmonically in time, but the frequency of oscillations is small and the inertia effects can be neglected. 3) For the sake of simplicity and compactness the problem formulation is made in a space of complex functionals. 4) Two scales of time are introduced denoted by t and τ, the first is associated with load slow oscillations (period counted in seconds) while the second is associated with tissue structure evolution (counted in days).

Basic relations and optimization problem

The constitutive viscoelastic relation:

$$\tilde{\sigma}_{ij}(\mathbf{x},\tau,t) = \int_0^t G_{ijkl}(\mathbf{x},\tau,t-S)\,\frac{d\tilde{\epsilon}_{kl}(\mathbf{x},\tau,S)}{dS}\,dS \tag{1}$$

where G_{ijkl}, $\tilde{\epsilon}_{kl}(\mathbf{x},\tau,t)$ and $\tilde{\sigma}_{ij}(\mathbf{x},\tau,t)$ denote relaxation function, stress and strain fields respectively, t represents time and τ is a real-valued parameter. For the steady-state harmonic oscillations we can assume that

$$\tilde{\epsilon}_{kl}(\mathbf{x},\tau,t) = \epsilon_{kl}\,(\mathbf{x},\tau)\,e^{\imath\omega t}\,, \qquad \tilde{\sigma}_{ij}(\mathbf{x},\tau,t) = \sigma_{ij}\,(\mathbf{x},\tau)\,e^{\imath\omega t}\,. \tag{2}$$

In the above equations $\imath = \sqrt{-1}$, ω is a circular frequency of oscillations while $\epsilon_{kl}(\mathbf{x},\tau)$ and $\sigma_{ij}\,(\mathbf{x},\tau)$ denote complex strain and stress fields. After time elimination we obtain from equation eq.1,

$$\sigma_{ij}(\mathbf{x},\tau) = G^*_{ijkl}(\mathbf{x},\omega)\,\epsilon_{kl}(\mathbf{x},\tau)\,, \;\; G^*_{ijkl}(\mathbf{x},\tau) = k(\mathbf{x},\tau)\,[G'_{ijkl}(\omega) + \imath\,G''_{ijlk}(\omega)]\,, \tag{3}$$

$$\sigma_{ij}(\mathbf{x},\tau) = \sigma^R_{ij}(\mathbf{x},\tau) + \imath\,\sigma^I_{ij}(\mathbf{x},\tau)\,, \;\; \epsilon_{ij}(\mathbf{x},\tau) = \epsilon^R_{ij}(\mathbf{x},\tau) + \imath\,\epsilon^I_{ij}(\mathbf{x},\tau)\,, \tag{4}$$

where $\dot{k} = \frac{dk(\mathbf{x},\tau)}{d\tau}$ represents the control function responsible for bone micro-structure. Complex strain energy is

$$U(\tau,\omega) = \frac{1}{2}\int_V \sigma_{ij}(\mathbf{x},\tau)\epsilon_{ij}(\mathbf{x},\tau) = U^R(\tau,\omega) + \imath\,U^I(\tau,\omega)\,. \tag{5}$$

Comparison functional and associated objective functional can be defined respectively as,

$$C(\tau,\omega,k) = (U^R)^2 + (U^I)^2\,, \qquad G(\tau,\omega,\dot{k}) = \frac{dC(\tau,\omega)}{d\tau}\,. \tag{6}$$

Global and local constraints,

$$\int_V \dot{k}(\mathbf{x},\tau)\, dV = A(\tau)\ , \quad \int_V \dot{k}^2(\mathbf{x},\tau)\, dV = B(\tau)\ , \tag{7}$$

$$k_{min}(\mathbf{x},\tau) \le k(\mathbf{x},\tau) \le k_{max}(\mathbf{x},\tau)\ , \quad \hat{k}_{min}(\mathbf{x},\tau) \le \dot{k}(\mathbf{x},\tau) \le \hat{k}_{max}(\mathbf{x},\tau) \tag{8}$$

Adaptation relations

The global and local constraints can be attached to the objective functional by means of Langrange multipliers. To obtain the functional relation between the unknown control function $\dot{k}$ and actual mechanical state of tissue the hypothesis of optimal response can be used. From the stationary conditions of the objective functional G the set of mathematical relations follow, among them the "adaptation law" controlling the adaptation behavior of bone.

Example

In the previous paper by the author the trabecular adaptive material composed of a large number of elastic beams was discussed, see [4]. In the present example similar, but viscoelastic material is considered. In this case for a simple constitutive viscoelastic relation in the form

$$p_0 \tilde{\sigma}_{ij}(\mathbf{x},\tau,t) + p_1 \frac{\partial \tilde{\sigma}_{ij}(\mathbf{x},\tau,t)}{\partial t} + p_2 \frac{\partial^2 \tilde{\sigma}_{ij}(\mathbf{x},\tau,t)}{\partial t^2} = q_0 \tilde{\epsilon}_{ij}(\mathbf{x},\tau,t) + q_1 \frac{\partial \tilde{\epsilon}_{ij}(\mathbf{x},\tau,t)}{\partial t} + q_2 \frac{\partial^2 \tilde{\epsilon}_{ij}(\mathbf{x},\tau,t)}{\partial t^2}\ , \tag{9}$$

the stimulus $S(\mathbf{x},\tau)$ for known values of stress amplitude $\sigma_0(\mathbf{x},\tau)$ and variable k can be expressed in the following form,

$$S(\mathbf{x},\tau) = \frac{1}{2} \frac{\sigma_0^4(\mathbf{x},\tau)\, p_0^2 + (p_1^2 - 2p_0\, p_2)\, \omega^2 + p_2^2 \omega^4}{k^3(\mathbf{x},\tau)\, q_0^2 + (q_1^2 - 2q_0\, q_2)\, \omega^2 + q_2^2\, \omega^2} \tag{10}$$

The relation between stimulus and circular frequency of periodic load for different sets of parameters is displayed in figure Fig.1 while in Fig.2 the result of computer simulation of bone adaptation is presented. The three pictures respect to the initial homogeneous situation, the result of bone adaptation to indicated external load, and the effect of bone remodeling after prosthesis implantation.

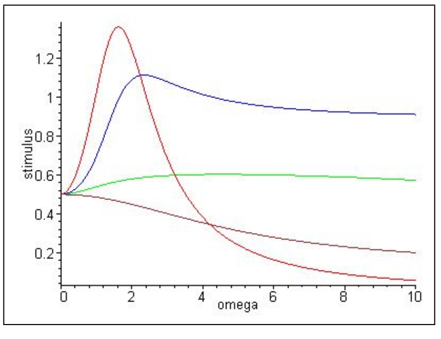

Fig. 1

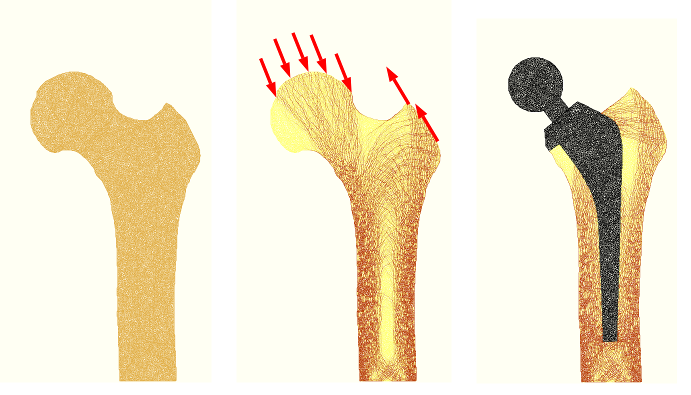

Fig. 2

CONCLUSIONS

The proposed approach enables derivation of different bone adaptation laws with viscoelastic effects included. It results from this derivation that the stimulus is frequency dependent and its functional relation on the frequency depends on the constitutive material model applied. For specific models the stimulus increases to some maximal value and then decreases to zero with growing to infinity frequency of load oscillations. The proposed theoretical approach may offer an attractive tool in investigations of frequency effect on bone remodeling but it needs more theoretical research and an experimental confirmation.

References

[1] Lanyon L.E., Rubin C.T. *J. of Biomechanics* **12**:897–905, 1984.
[2] Taber L.A.: *Appl. Mech. Rev.* **48**, 8:487–545, 1995.
[3] Prendergast P.: *J. of Biomechanics*, **27**:1067–1078, 1983.
[4] Lekszycki T.: *J. of Theoret. and Appl. Mech.*, **3**, 37:607–624, 1999
[5] Lekszycki T.: *Meccanica*, **37**:343–354, 2002

Acknowledgment

The present work was partly supported from the EU Research Project No. QLRT-1999-02014 (MIAB)