QUASI-GEOSTROPHIC DYNAMOS

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Summary Taking advantage of the properties of liquid metals and of rapidly rotating flows, we are able to compute dynamos at high Reynolds number ($Re > 10^5$) and low magnetic Prandtl number ($Pm < 10^{-2}$)

We developed a numerical model that uses a quasi-goestrophic approximation to compute the flow (without subgrid scale model), leading to two-dimensional equations. The induction equation for the magnetic field is fully resolved in 3D, in a sphere. This approach proves quite efficient for low magnetic Prandtl number and suitable flows, for which there is a scale separation between magnetic field and velocity field, allowing to compute the magnetic field on a coarser grid and with larger time steps than for the velocity field. We show results of these calculations applied on the turbulent flow produced by the destabilization of a Stewartson shear layer.

INTRODUCTION

The magnetic field of the Earth is produced by a dynamo effect in the metallic liquid core of our rotating planet. Many efforts have been done successfully in the last decade to describe the mechanism of a self induced magnetic field with experimental models and numerical simulations. Both approaches have limitations. No experiment has been done in rotation while rotation is thought as a key ingredient by geophysicists to explain the geometry and the amplitude of the geomagnetic field. All numerical models [4, 5, 6], have introduced the Coriolis force in solving the Navier-Stokes equation, but the prescribed magnetic Prandtl number ($P_m = \nu/\eta$, where ν is the kinematic viscosity and η the magnetic diffusivity) is of order unity, due to current computer capabilities. However, liquid metals exhibit magnetic Prandtl number lower than 10^{-5} in planetary cores [7]. This low value of P_m may indicate a separation, in terms of scales and frequencies, between the velocity and magnetic fields in a metallic dynamo.

NUMERICAL MODEL

We propose an approach that aims to compute very low magnetic Prandtl number dynamo taking advantage of the geostrophic behavior of the velocity field. For very low Ekman number ($E = \nu/\ \Omega R^2$, where Ω is the rotation rate of the spherical container, and R its radius), a quasi-geostrophic (QG) approach model correctly the flow in a rapidly rotating sphere [1, 2, 3]. It consists of the integration of the Navier-Stokes equation along the axis of rotation. Even if the numerical resolution is done with a pseudo current function in the equatorial plane (2D), the top and bottom boundary effect are present through β and Ekman pumping effects in the Coriolis term. We compute the QG flow in the equatorial plane with a fine spatio-temporal resolution, the velocity is extrapolated to a coarse 3D spherical grid where the induction equation is solved. Then, the z-integrated associated Lorentz forces is used to compute the evolution of the velocity field. Even if there is some computation time spent in the conversion between grids, this method is efficient if the scale separation is large enough.

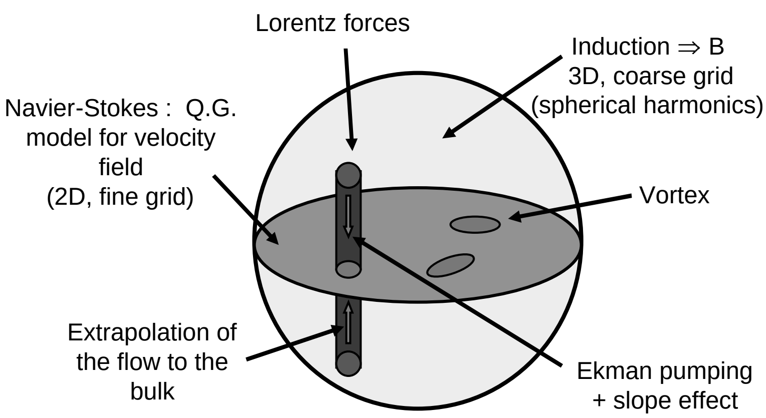

Diagram showing the principles of our quasi-geostrophic dynamo model

TEST CASE : THE STEWARTSON DYNAMO

The forcing

In order to demonstrate the validity of this approach, we decide to apply it to a simple case. Instead of a thermal convective flow where the heat equation has to be solved, we model the flow associated wit the instabilities of an internal geostrophic shear layer. This layer, known as the Stewartson layer, is produced by a differentially rotating inner core in a rotating sphere and consists of two nested viscous shear layers. For large Ro number ($Ro = \Delta\Omega/\Omega$, where $\Delta\Omega$ is the differential rate of rotation of the inner core), the Stewartson layer may become unstable and generate Rossby waves [8]. For even larger Ro, the flow becomes turbulent. However this turbulence is strongly influenced by the global rotation of the system and is large-scales dominated, with a steep k^{-5} spectrum. This suggest that we can properly neglect the induction due to small scales.

Kinematic dynamo results

For the kinematic dynamo study, we don't compute the Lorentz force, so that the evolution of the flow remains purely hydrodynamic. Some dynamo experiments are reported on the following table. There does not seem to be a limit in Pm or E for driving a dynamo. The only restriction is that the Rossby number remains small enough so that the velocity field can be considered as QG. We will also report some specific features of these dynamos.

E	Ro	Pm	Re
10^{-5}	0.04	5	100
10^{-6}	0.013	0.6	1000
10^{-6}	0.04	0.3	10000
10^{-7}	0.015	0.03	100000
10^{-8}	0.02	0.005	400000

Kinematic dynamos runs for various E,Ro and Pm input parameters. Re is an output parameter.

Lorentz force back-reaction

With the Lorentz force, the magnetic field acts back on the velocity field. However we can show that in the regime of small Rossby numbers, the Elsasser number (measuring the ratio of the Lorentz force by the Coriolis force) remains small, insuring that the Proudman-Taylor constraint is not violated.

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