

## LOW CYCLE FATIGUE BASED ON UNILATERAL DAMAGE EVOLUTION

Artur Ganczarski

Cracow University of Technology, Institute of Applied Mechanics, PL 30-864 Kraków, Poland

*Summary* The present paper deals with a modelling of low cycle fatigue of AISI 316L stainless steel with aid of kinetic law of unilateral damage evolution. Results of numerical simulation are compared and verified in order to achieve the best agreement with the experimental data. Detailed quantitative and qualitative analysis of obtained solutions confirms necessity and correctness of an application of continuous microcrack closure/opening effect.

### INRODUCTION

The most frequent approach to the low cycle fatigue, in case when a loading is periodic strain-controlled of constant amplitude, consists in taking into account the following assumptions: the material becomes perfectly plastic during first cycle, the variation of damage is neglected for the integration over one cycle and the stain-damage relations are identical both under tension and compression. These allow to simplify calculations of damage accumulation per one cycle and give linear dependency of damage with number of cycles, finally leading to the Manson-Coffin law of low cycle fatigue [4, 8, 9, 10, 11]. On the other hand, the more refined approaches to the low cycle fatigue presented in [1] and [7], based on the kinetic theory of damage evolution and the Gurson-Tvergaard-Needleman model of damage incorporated isotropic hardening, respectively, are able to predict some qualitative phenomena of damage accumulation, crack initiation and fracture only approximately since they do not account for the unilateral damage evolution.

### GENERAL EQUATIONS

When a material is subjected to a cyclic loading at high values of stress or strain, damage develops together with cyclic plastic strain. If the material is strain loaded, the damage induces a drop of the stress amplitude and the decrease of the elasticity modulus. Damage evolution theory, suitable to the model such phenomenon is due to Lemaitre and Chaboche [7, 8]. The fundamental assumption is that, in case of low cycle fatigue, the damage is related to the accumulated plastic strain  $p$ . In the present formulation, as in the original one, the potential of dissipation is a sum of two parts, the first referring to Mises yield condition, where damage coupling acts only by the effective stress, and the other associated with the kinetic law of damage evolution. However, two separate damage mechanisms corresponding to isotropic transgranular microcracks growth and unilateral growth intergranular microcracks require application of two independent scalar damage variables. The first isotropic damage variable  $D_s$  acts on the deviatoric stress and the second unilateral damage variable  $D_v$  acts on the volumetric stress. Unilateral damage requires separate definitions of the effective stress, which allow to distinguish the case of tension (+), when microcracks remain open, and the case of compression (-), when they are partly closed, the effective stress takes final form valid for one dimension case

$$\tilde{\sigma}^{\pm} = \frac{\sigma}{3(1-D_s)} + \frac{2\sigma}{3(1-D_v h)} \quad h = \begin{cases} 1 & \text{for } \sigma > 0 \\ h_c & \text{for } \sigma < 0 \end{cases} \quad (1)$$

where  $h \in [h_c, 1]$  stands for damage closure/opening parameter. Application of the classical formalism of associated plasticity to the potential of dissipation dependent of such defined effective stress Eq.(1) leads to the following form of the kinetic damage law written for one dimension case

$$\frac{dD_s}{dp} = \frac{\sigma^2}{3ES_s(1-D_s)^2} \exp(-ap) \quad \frac{dD_v}{dp} = \frac{\sigma^2(1-D_s)}{6ES_v(1-D_v)^3} H(p-p_D) \quad (2)$$

In order to obtain smooth transition of the strain-stress relation between the elastic range and the nonlinear plastic range the modified Ylinen approximation for loading cycles is applied

$$\frac{d\sigma}{d\varepsilon} = \tilde{E}^{\pm} \frac{\tilde{\sigma}_0^{\pm} - |\sigma|}{\tilde{\sigma}_0^{\pm} - c|\sigma|} \quad \text{for loading} \quad \frac{d\sigma}{d\varepsilon} = \tilde{E}^{\pm} \quad \text{for unloading} \quad (3)$$

where the effective damage modulus of elasticity and the effective asymptotic yield stress of Ylinen's model become functions of damage

$$\frac{\tilde{E}^{\pm}}{E} = \frac{\tilde{\sigma}_0^{\pm}}{\sigma_0} = \frac{(1-D_s)(1-D_v h)}{1 - \frac{2}{3}D_v h - \frac{1}{3}D_s} \quad (4)$$

In case of unloading path, introduced definition of the effective stress Eq.(1), accounting for the crack closure/opening effect, leads to the linear relation between stress and strain that inclination is described by modulus  $\tilde{E}(D)$ . The defect of such a model is the switch from the part of path described by  $\tilde{E}^+$  to the part of path characterised by the modulus of elasticity  $\tilde{E}^-$ . The real materials, however, do not exhibit such bilinear paths. Therefore, the more realistic model,

closer to real material behaviour, seems to be a proposal of curve-linear path, suggested in [5], inclination of which depends on percentage of microcrack closure. In the simplest case, the concept of continuous crack closure/opening consists in the replacement of parameter  $h$  by a function  $h(\sigma)$  such that for one dimension state of stress is [3]

$$h(\sigma) = h_c + (1 - h_c) \langle \sigma \rangle / \sigma_b \quad (5)$$

Finally, simplified description of the neck mechanism, directly preceding failure, taking advantage of theory by Davidenkov-Spiridonova leads to the following 3D extensions of Ylinen's law

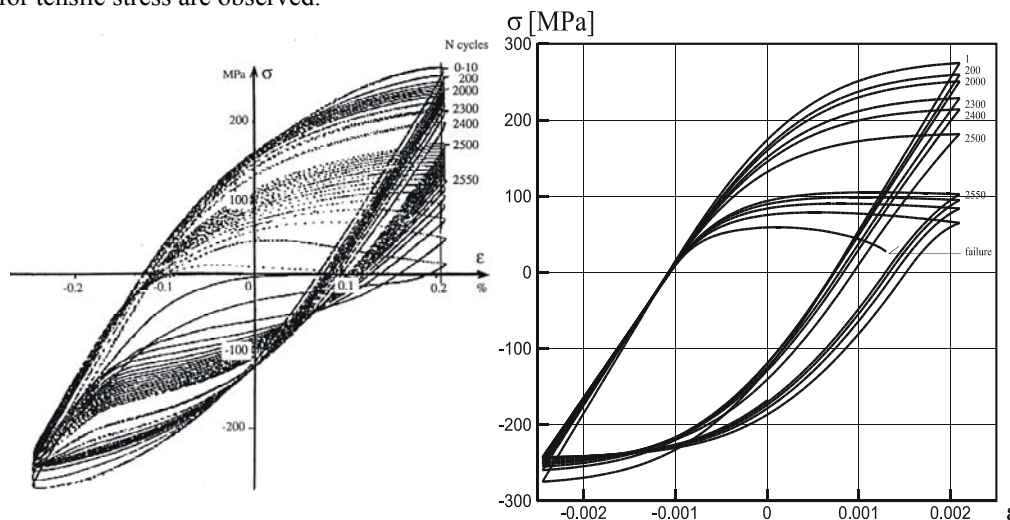
$$\frac{d\sigma}{d\varepsilon} = E^* \frac{\sigma_0^* - |\sigma|}{\sigma_0^* - c|\sigma|} \quad \frac{E^*}{\tilde{E}^\pm} = \frac{\sigma_0^*}{\tilde{\sigma}_0^\pm} = \frac{1 + 0.25b\sqrt{\langle \varepsilon \rangle}}{1 + 0.5b\sqrt{\langle \varepsilon \rangle}} \quad (6)$$

and kinetic damage law

$$\frac{dD_s}{dp} = \frac{\sigma^2}{3\sqrt{2}ES_s(1-D_s)^2} \frac{\exp(-ap)}{(1+0.25b\sqrt{\langle \varepsilon \rangle})} \quad \frac{dD_v}{dp} = \frac{\sigma^2(1-D_s)}{6\sqrt{2}ES_v(1-D_v)^3} \left( \frac{1+1.5b\sqrt{\langle \varepsilon \rangle}}{1+0.25b\sqrt{\langle \varepsilon \rangle}} \right)^2 H(p-p_D) \quad (7)$$

## RESULTS

A systems of three ordinary differential equations Eqs (2, 3) or Eqs (6, 7) are numerically integrated using fourth-order Runge-Kutta technique with adaptive stepsize control. The numerical simulation with presented model of continuous crack closure/opening effect exhibits not only quantitative but also qualitative excellent correctness in comparison with experimental results. The smooth transition between tensile and compression as well as formation of the stress instability for tensile stress are observed.



Low cycle fatigue stress strain pattern for AISI 316L: Dufailly's experiment (left) and numerical simulation (right)

## CONCLUSIONS

The model presented in this paper demonstrates necessity and correctness of the application of continuous microcrack closure/opening effect in kinetic law of damage evolution in order to model low cycle fatigue of AISI 316L stainless steel.

## References

- [1] Brocks W., Steglich D.: Damage models for cyclic plasticity. *Key Engineering Materials Vols.*: 389-398, 2003.
- [2] Dufailly J., Lemaître J.: Modeling of low cycle fatigue. *Int. J. Damage Mechanics*: 4, 153-170, 1995.
- [3] Foryś P., Ganczarski A.: Modelling of microcrack closure effect. *Proc. Int. Symp. Anisotropic Behaviour of Damaged Materials*, 1983 (on CD).
- [4] Garion C., Skoczeń B.: Combined model of strain induced phase transformation and orthotropic damage in ductile materials at cryogenic temperatures. *Int. J. Damage Mech.*, 2003 (in print).
- [5] Hansen N.R., Schreyer H.L.: Damage deactivation, *Trans. ASME*: 62, 450-458, 1995.
- [6] Ladeveze P.: On an anisotropic damage theory. in *Failure criteria of structural media*, CNRS Int. Coll. No 351, Villard-de-Lans, ed. Boehler, Balkema, Rotterdam, 1983.
- [7] Lemaître J.: *A course on damage mechanics*, Springer-Verlag, 1992.
- [8] Lemaître J., Chaboche, J.-L.: *Mécanique des matériaux solides*, Bordas, Paris, 1985.
- [9] Skoczeń B.: Generalisation of the Coffin equations with respect to the effect of large mean plastic strain, *ASME Journal of Eng. Mat. Techn.*, 118, 1996.
- [10] Skoczeń B.: *Material and structural stability issues at cryogenic temperatures*, Habilitation thesis, Cracow University of Technology, 88, 2002.
- [11] Skrzypek J.J., Ganczarski A.: *Modeling of material damage and failure of structures*, Springer-Verlag Berlin Heidelberg, 1999.