Non-Uniform Flow Hydrodynamics of Deformable Shapes

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This contribution is motivated by the recent works of Moffatt and Sellier (2002), Sellier (2003), Shin and Kang (2002) and Kang et al. (2002). Moffatt and Sellier examined the low Reynolds number migration of an insulating rigid particle in a viscous liquid metal which is subject to uniform electric and magnetic fields E and B in the Stokes realm. Complementary to these studies, Shin and Kang considered the effect of a uniform magnetic field on the shape of a deformable bubble embedded in an homogeneous weakly viscous conducting fluid in a uniaxial straining flow. In the present paper, we consider a much broader class of practical multiphase problems, namely the trajectories of arbitrary 3-D deformable shapes which move with six degrees of freedom in a conducting viscous liquid in a pure ambient shearing flow. In order to obtain analytic expressions for the pressure-induced force and moment exerted on the moving body, we assume for simplicity that the linear ambient flow field can be represented by

$$\boldsymbol{u}(\boldsymbol{r},t) = \boldsymbol{V}(t) + \mathsf{G}(t)\boldsymbol{r} + \frac{1}{2}\,\boldsymbol{\omega}(t) \times \boldsymbol{r} \tag{1}$$

where $\boldsymbol{V}$ is a uniform stream, $\mathsf{G}(t)$ is a second-rank symmetric tensor with zero trace and $\boldsymbol{\omega}(t) = \nabla \times \boldsymbol{u}$ is the ambient vorticity field. The pressure field is determined from the governing Navier-Stokes equation which includes the Lorentz term, i.e.,

$$\rho\frac{D\boldsymbol{u}}{Dt} = -\nabla p + \boldsymbol{J} \times \boldsymbol{B} + \nu\nabla^2\boldsymbol{u} \tag{2}$$

The time-dependent magnetic field $B(t)$ which evolves with time according to Maxwell's equation also depends on the ambient vorticity. The electric current density in the case of a low magnetic Reynolds number can be obtained from Ohm's law as

$$\boldsymbol{J} = \sigma(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}) \tag{3}$$

where $\boldsymbol{E}$ is the ambient electric field and σ is the electric conductivity. It is next assumed that a general deformable moving body is impulsively introduced into the ambient flow field and our goal is to first derive analytic expressions for the forces and moments acting on the body. Limiting the analysis to clean bubble surfaces (Magnaudet and Eames, 2000), it can be rigorously shown that at least during a short time after introducing the body, viscous effects can be neglected and the induced velocity field in the viscous conducting field, (due to the motion of the body) is essentially of an irrotational nature. The analytic expressions for the pressure loads can be next integrated so as to render the complete trajectories of the deformable shape. If the ambient fluid is non-conductive, the present analysis reduces to that given in Miloh (2003, 2004).

References

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