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A PERTURBATION METHOD FOR NONLINEAR VIBRATIONS OF STRUCTURES

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Summary A perturbation theory for the large amplitude vibration analysis of general, statically loaded imperfect structures is used to investigate the dependence of the natural frequencies of structures on finite vibration amplitudes, initial geometric imperfections, and a nonlinear static deformation. An extension to a multi-mode analysis is presented. Characteristic results of the buckling and nonlinear vibration analysis of isotropic and composite shells illustrate the capabilities of the computational modules developed.

INTRODUCTION

A perturbation theory for the large amplitude vibration analysis of general, statically loaded imperfect structures is presented. The method is based on a perturbation expansion for both the frequency parameter and the dependent variables. The theory developed forms an extension of earlier work in [4] and [5], and makes it possible to investigate the dependence of the natural frequencies of structures on finite vibration amplitudes, initial geometric imperfections, and a nonlinear static deformation (orthogonal to the axisymmetric fundamental state).

STATIC AND DYNAMIC ANALYSIS

Using D'Alembert's principle to introduce inertial loading, the variational equation of motion (dynamic equilibrium) can be formulated as follows,

$$M(\ddot{\mathbf{u}})\cdot\delta\mathbf{u}+\sigma\cdot\delta\varepsilon=\mathbf{q}\cdot\delta\mathbf{u} \tag{1}$$

where $\mathbf{u}$, ε, and σ denote the generalized displacement, strain, and stress, respectively. These field variables can be interpreted as vector functions with variables appropriate to the problem. Further, M is the generalized mass operator, $\mathbf{q}$ is the applied load, and $\ddot{(\,)}=\partial^2(\,)/\partial t^2$, where t denotes time. In Eq. (1) $\mathbf{a}\cdot\mathbf{b}$ is the virtual work of stresses or loads $\mathbf{a}$ acting through strains or displacements $\mathbf{b}$, integrated over the entire structure for kinematically admissible variations $\delta\mathbf{u}$. In addition, we have the strain-displacement relation

$$\varepsilon=L_1(\mathbf{u})+\frac{1}{2}L_2(\mathbf{u})+L_{11}(\bar{\mathbf{u}},\mathbf{u}) \tag{2}$$

where $\bar{\mathbf{u}}$ is an initial geometric imperfection, and where L_1 and L_2 are homogeneous linear and quadratic functionals, respectively, and L_{11} is a homogeneous bilinear functional [4, 5].

The situation of a dynamic state on a nonlinear static state is analysed for the case that a natural frequency corresponds to a single vibration mode. Assuming that the structure is vibrating in a deformed configuration due to a static loading (the 'static' state), the variables can be written as a superposition of two states. The static state response in turn is assumed to consist of a (nonlinear) fundamental ('trivial') state and a nonlinear buckling ('nontrivial' or 'orthogonal') state. The nonlinearity of the buckling state is taken into account via a perturbation expansion of the buckling variables. The equations governing the equilibrium of the fundamental static state, the (nonlinear) buckling state, and the dynamic state, respectively, can be derived systematically.

The first-order state equation forms an eigenvalue problem for the unknown natural square frequency ω_0^2. The relation of the frequency squared to the vibration amplitude and the effective imperfection amplitude is derived via a contraction procedure, i.e., the first-order equilibrium equation at the critical point (which can be seen as an orthogonalization condition for the higher modes with respect to the critical mode) is used as a constraint to eliminate the third-order variables.

DYNAMIC MULTI-MODE ANALYSIS

If a natural frequency corresponds to more than one vibration mode, a multi-mode analysis can be performed, analogous to the analysis in [2] for buckling mode interaction. Supposing that there are M vibration modes, denoted as $\mathbf{u}_{d_i}$, with corresponding natural frequency squared $\omega^2=\omega_{0_i}^2$ and amplitude ξ_i ($i=1,2,\ldots,M$), the dynamic state displacement field for a 'perfect' unloaded structure can be written as

$$\mathbf{u}_d=\xi_i\mathbf{u}_{d_i}+\xi_i\xi_j\mathbf{u}_{d_{ij}}+\ldots \tag{3}$$

with corresponding expressions for the stress and strain fields,

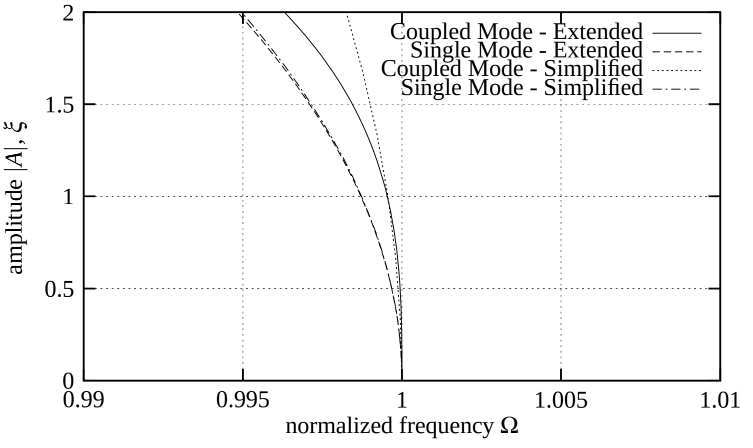

Figure 1. *Amplitude-frequency curves for Single Mode and Coupled Mode nonlinear vibrations of isotropic shell. Comparison between Simplified Analysis (vibration mode amplitude $|\bar{A}|$) and present Extended Analysis (vibration mode amplitude ξ); $\Omega = \omega/\omega_{0_1}$.*

$$\sigma_d = \xi_i \sigma_{d_i} + \xi_i \xi_j \sigma_{d_{ij}} + \dots \tag{4}$$

$$\varepsilon_d = \xi_i \varepsilon_{d_i} + \xi_i \xi_j \varepsilon_{d_{ij}} + \dots \tag{5}$$

In this case nonlinear amplitude-frequency relations are obtained of the following form,

$$\xi_I [1 - \frac{\omega^2}{\omega_{0_I}^2}] + A_{ijI} \xi_i \xi_j + B_{ijkI} \xi_i \xi_j \xi_k = 0, \quad I = 1, 2, \dots, M \tag{6}$$

where the coefficients A_{ijI} and B_{ijkI} depend on the first-order and second-order stress and strain fields, respectively.
The perturbation theory developed is applied to the nonlinear (large amplitude) vibration problem of anisotropic cylindrical shells including edge effects. The starting point of the analysis are the Donnell-type differential equations of an anisotropic circular cylindrical shell. An axisymmetric fundamental state is included in the formulation. The first-order state problem constitutes an eigenvalue problem for the unknown eigenfrequencies and vibration modes. The associated higher-order state problems are response problems with coefficients that depend on the solution of the first-order state problem.
Koiter's initial post-buckling and imperfection sensitivity theory deals with the dependence of the load parameter on the deflection and imperfection amplitudes in the case of static buckling problems. In [1], Koiter's theory has been applied to the buckling of composite cylindrical shells. The theory developed in the present paper is related to the work described in [1]. The perturbation approach has been applied to the single mode nonlinear vibrations of composite shells in [3].

RESULTS

Characteristic results of the buckling and nonlinear vibration analysis of isotropic and composite shells are presented, in order to illustrate the capabilities of the computational modules developed. In Figure 1 the amplitude - frequency curves of the coupled mode nonlinear vibrations of an isotropic shell are compared with the single mode results, for different levels of analysis complexity. In the present analysis (denoted as 'Extended Analysis') the specified ('simply supported') boundary conditions are taken into account accurately, while in the 'Simplified Analysis' these boundary conditions are satisfied only approximately.

References

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