NONLINEAR VIBRATIONS OF SHALLOW SHELLS AND THIN PLATES OF ARBITRARY SHAPE

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Summary Large amplitude flexural vibrations of thin plates and shallow shells with the complex plans are studied by the R-function method (RFM). The algorithm is based on the reducing the given problem to research of the nonlinear differential equation of Duffing type. Its factors are determined as a result of solving the boundary value problem by variational Ritz method. The sequences of coordinate functions were constructed by the R-function theory. Obtained results are compared with well-known from other works.

PROBLEM STATEMENT

For research of geometrically nonlinear free vibrations of isotropic shallow shells (neglecting tangential inertia) the equations [1] are used.

$$\rho\frac{12(1-\mu^2)}{Eh^2}\frac{\partial^2 w}{\partial t^2}+\nabla^4 w+\frac{12}{h^2}\left\{(k_x+\mu k_y)\frac{\partial u}{\partial x}+(\mu k_x+k_y)\frac{\partial u}{\partial y}+(k_x^2+k_y^2+2\mu k_x k_y)w+\frac{k_x+\mu k_y}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{k_y+\mu k_x}{2}\left(\frac{\partial w}{\partial y}\right)^2-\right.$$

$$\left.-\frac{\partial^2 w}{\partial x^2}\left[\frac{\partial u}{\partial x}+\mu\frac{\partial v}{\partial y}+(k_x+\mu k_y)w+\frac{1}{2}\left(\frac{\partial w}{\partial x}+\mu\frac{\partial w}{\partial y}\right)^2\right]-(1-\mu)\frac{\partial^2 w}{\partial x\partial y}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)-\frac{\partial w}{\partial y}\left[\mu\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+(\mu k_x+k_y)w+\frac{1}{2}\left(\frac{\partial w}{\partial y}+\mu\frac{\partial w}{\partial x}\right)^2\right]\right\}=0, \quad (1)$$

$$A\vec{U}=L_k(w)+L(w), \quad (2)$$

where
$$A\vec{U}=\begin{bmatrix}\frac{\partial^2}{\partial x^2}+\frac{1-\mu}{2}\frac{\partial^2}{\partial y^2} & \frac{1+\mu}{2}\frac{\partial^2}{\partial x\partial y}\\ \frac{1+\mu}{2}\frac{\partial^2}{\partial x\partial y} & \frac{\partial^2}{\partial y^2}+\frac{1-\mu}{2}\frac{\partial^2}{\partial x^2}\end{bmatrix}\begin{bmatrix}u\\ v\end{bmatrix},\quad L_k(w)=-\begin{bmatrix}(k_x+\mu k_y)\frac{\partial w}{\partial x}\\ (k_y+\mu k_x)\frac{\partial w}{\partial y}\end{bmatrix},\quad L(w)=-\begin{bmatrix}\frac{1}{2}(w_{,1}^2+\mu w_{,2}^2)_{,1}+\frac{1-\mu}{2}(w_{,1}w_{,2})_{,2}\\ \frac{1-\mu}{2}(w_{,1}w_{,2})_{,1}+\frac{1}{2}(w_{,2}^2+\mu w_{,1}^2)_{,2}\end{bmatrix}.$$

Here $\vec{U}=(u,v)$, w are displacements of a shallow shell median surface in directions x,y,z; k_x,k_y are curvatures of the shallow shell element, E, h, ρ, μ are the modulus of elasticity, thickness of shell element, density of a material and Poisson's ratio, respectively. The equations (1-2) should be jointed with appropriate boundary and initial conditions.

THE METHOD OF SOLUTION

The deflection $w(x,y,t)$ is presented as

$$w(x,y,t)=y_1(t)W_1(x,y), \quad (3)$$

where $W_1(x,y)$ is the eigenfunction, appropriate to the basic linear frequency ω_L. Substituting (3) into (2), one can receive the system of equations. Its solution may be considered as

$$\vec{U}(x,y,t)=y_1(t)\vec{U}_1(x,y)+y_1^2(t)\vec{U}_2(x,y). \quad (4)$$

Here $\vec{U}_1=(u_1,v_1)$ is the solution of system $A\vec{U}_1=L_k(W_1)$, and $\vec{U}_2=(u_2,v_2)$ is the solution of system $A\vec{U}_2=L(W_1)$.
After substituting the known functions u_1,v_1,u_2,v_2,W_1 in (1) and using the Galerkin procedure, one will receive the nonlinear differential equation on time

$$y''_{1t}(t)+\omega_L^2\left(y_1(t)+\gamma y_1^2(t)+\beta y_1^3(t)\right)=0, \quad (5)$$

where factors are double integrals from vector functions $\vec{U}_1,\vec{U}_2,W_1$.
Integrating the given equation by Lindshtedt method [2] or Galerkin procedure [1] (here the orthogonalization is carried out only on a quarter of the period of vibrations), it is possible to receive the period of nonlinear vibrations in the first case, and obvious dependence between nonlinear and linear natural frequencies in the second case, i.e.

$$\upsilon=\frac{\omega}{\omega_L}=\sqrt{1+\frac{8}{3\pi}\gamma A+\frac{3}{4}\beta A^2} \quad \text{(for plates } \gamma=0\text{)}. \quad (6)$$

For shallow shells and plates of arbitrary shape there is a problem to define natural frequencies and the appropriate eigenfunctions, and also displacements vector $\vec{U}_2$. In the present work this problem is carried out by the R-function method (RFM) [3] coupled with variational Ritz method.

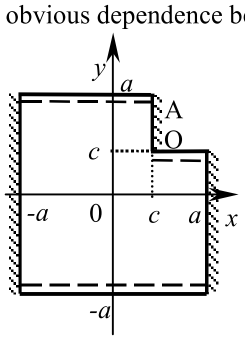


Fig. 1

NUMERICAL RESULTS

Let's consider a problem about free nonlinear vibrations of a plate (fig.1) with the following boundary conditions: the plate is clamped along $x=\pm a, x=c$ and simply supported along

$y = \pm b,\ y = c$. The natural frequency and the natural mode of vibrations can be found as a result of solving of the problem considering the minimum of the Lagrangian [4] on set of the functions, which are the linear combination of coordinate sequences $\{u_i\},\{v_i\},\{W_{1i}\}$, satisfying given boundary conditions. The technique of construction of the sequences of coordinate functions is described by V.L. Rvachev in [3] in detail enough. Then the solution of linear vibration problem of shallow shell can be as follows

$$u \approx \sum_{i=1}^{N_1} a_i u_i(x,y),\ v \approx \sum_{i=N_1+1}^{N_2} a_i v_i(x,y),\ W_1 \approx \sum_{i=N_2+1}^{N_3} a_i W_{1i}(x,y).$$

For given example the sequences of coordinate functions are $u_i = \omega\varphi_{1i}, v_i = \omega\varphi_{2i}, W_{1i} = \omega\omega_1\varphi_{3i}$, where $\{\varphi_{ji}\}(j = 1,2,3; i = 1,...N_3)$ are any complete sequences of functions, for example, power polynomials, $\omega = 0$ is the equation of the whole area's boundary and $\omega_1 = 0$ is the equation of the clamped part of the boundary. Knowing the natural vector appropriate to the basic frequency ω_L and solving system $A\vec{U}_2 = L(W_1)$, functions u_2, v_2 are determined. The last system is solved also by Ritz method. The calculation was carried out at total number of coordinate functions 75, that corresponds to 4-th degrees power polynomials approximating u and v, and 8-th degrees polynomial approximating W_1. Testing of the suggested method and the appropriate software was executed for a plate shown in fig.1 if $c = 0$. The obtained results are presented on fig.2. The numerical results for square simply supported plate are shown on fig.3*a* and for square clamped one are at fig.3*b*. For the complex shape plate (fig.1) the influence of cut out depth c/a upon frequencies ratio υ were investigated. Obtained results are presented on fig.2. Whenever possible numerical results are compared with similar of other works. It allows to confirm the reliability of suggested method.

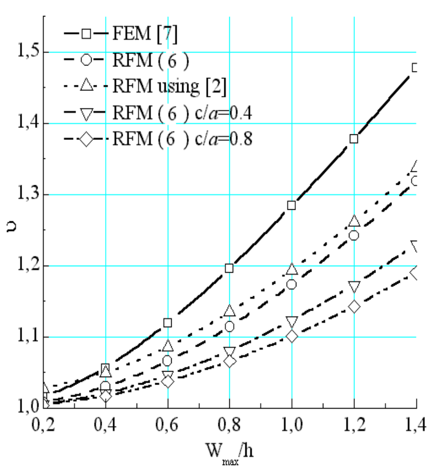
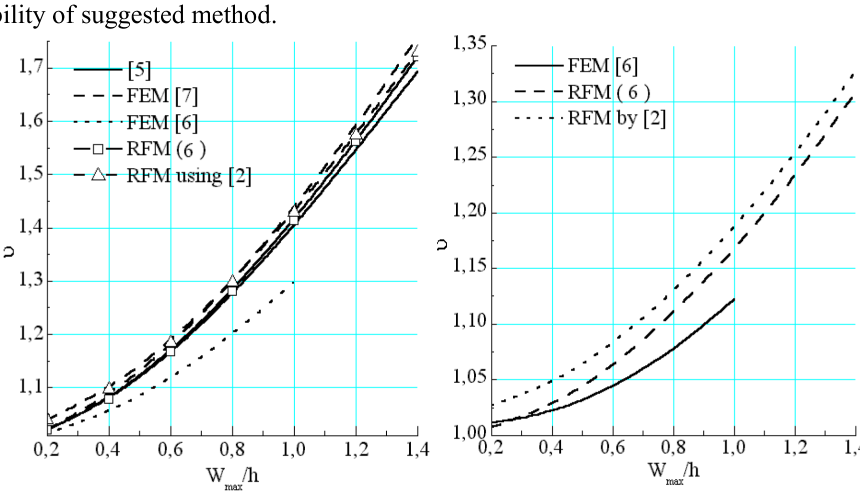


Fig.2. Effect of amplitude upon frequency for plate fig.1



a). simply supported *b*). clamped

Fig. 3(*a,b*). Effect of amplitude upon frequency for square plate with various types of boundary conditions

CONCLUSIONS

The suggested method for an investigation of large amplitude free vibration of shallow shells and thin plates and corresponding software are effective due to the R-function theory. They can be used for shallow shells of arbitrary shape and various types of boundary conditions research.

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