FE-INVESTIGATIONS ON SHEAR LOCALIZATIONS IN GRANULAR BODIES WITHIN HYPOPLASTICITY

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Summary A spontaneous shear localization in granular bodies is investigated with a finite element method based on a hypoplastic constitutive model. To simulate properly the formation of shear zones, a hypoplastic model was extended by polar, non-local and gradient terms to take into account a characteristic length of the microstructure. Two different rate boundary value problems were numerically analyzed with an extended model.

INTRODUCTION

Localization of deformation in the form of narrow zones of intense shearing is a fundamental phenomenon in granular materials. It can develop in granular bodies during processes of flow or shift of objects with sharp edges against granular materials. It can occur spontaneously inside of granular materials or can be induced along walls of stiff structures at granular bodies. An understanding of the mechanism of the formation of shear zones is important since they act as a precursor to ultimate soils failure.
Classical FE-analyses of shear zones are not able to describe properly both the thickness of localization zones and distances between them since they suffer from a spurious mesh sensitivity (its size and alignment). The rate boundary value problem becomes ill-posed, i.e. the governing differential equations of motion change the type by losing ellipticity for static and hiperbolicity for dynamic problems. Thus, the localization is reduced to zero-volume zones. To overcome this drawback, classical constitutive models require an extension in the form of a characteristic length to regularize the rate boundary value problem and to take into account microscopic inhomogeneities triggering shear localization (e.g. size and spacing of microdefects, grain size, fiber spacing). Different strategies can be used to include a characteristic length and to capture properly the post-failure regime (in quasi-static problems): polar models, non-local models, strain gradient models, and models with an artificial viscosity.
In this paper, a spontaneous shear localization in granular bodies was investigated with a finite element method based on a hypoplastic constitutive law extended by polar, non-local and gradient terms. The FE-analysis was performed with enhanced hypoplastic models for a specimen of dry sand for two different rate boundary value problems: vertical compression under constant lateral pressure and earth pressure of a retaining wall in conditions of plane strain. The numerical results were compared to corresponding laboratory tests carried out at the Karlsruhe, Cambridge and Tokyo University.

HYPOPLASTICITY

Hypoplastic constitutive models [1], [2] are an alternative to elasto-plastic ones for continuum modelling of granular materials. In contrast to elasto-plastic models, a decomposition of deformation components into elastic and plastic parts, yield surface, plastic potential, flow rule and hardening rule are not needed. The hypoplastic models describe the behaviour of so-called simple grain skeletons assuming that the macroscopic state can be sufficiently characterised by mean values of stress and void ratio. They describe the evolution of effective stress components with the evolution of strain components by a differential equation including isotropic linear and non-linear tensorial functions. The hypoplastic laws include also barotropy (dependence on pressure level), pycnotropy (dependence on density), dependence on the direction of deformation rate, dilatancy and contractancy during shearing with constant pressure, increase and release of pressure during shearing with constant volume, and material softening during shearing of a dense material. The feature of the model is a simple formulation and procedure for determination of material parameters with standard laboratory experiments [3]. The parameters are related to granulometric properties encompassing grain size distribution curve, shape, angularity and hardness of grains. Owing to that one set of material parameters is valid within a large range of pressures and densities. To simulate properly the formation of shear zones, a hypoplastic model was extended by polar, non-local and gradient terms to take into account a characteristic length of the microstructure.

EXTENDED HYPOPLASTICITY

The polar terms were introduced in a hypoplastic law with the aid of a polar (Cosserat) continuum. A Cosserat continuum takes into account two linked levels of deformation: micro-deformation at the particle level and macro-deformation at the structural level. Each material point has for the case of plane strain three degrees of freedom: two translational degrees of freedom and one independent rotational degree of freedom. The gradients of the rotation are connected to curvatures which are associated with couple stresses. It leads to a non-symmetry of the stress tensor and a presence of a characteristic length in the form of a mean grain size. The capability of a polar hypoplastic model has already been demonstrated in solving boundary value problems involving localization such as biaxial test, shearing of a narrow granular layer, silo filling and silo flow, furnace flow, footings and sand anchors [4]-[7]. A close agreement between calculations and experiments was achieved. A polar model has good physical grounds since it takes into account rotations and couple stresses which are observed during shearing but remain negligible during homogeneous deformation. A characteristic length is directly related to a mean grain diameter.

A non-local approach is based on spatial averaging of tensor or scalar state variables in a certain neighborhood of a given point (i.e. material response at a point depends both on the state of its neighborhood and the state of the point itself). To obtain a full regularization effect according to both the mesh size and mesh inclination, it is sufficient to treat non-locally only one internal constitutive variable (e.g. equivalent plastic strain in an elasto-plastic formulation or modulus of the deformation rate in a hypoplastic approach [8]) whereas the others can retain their local definitions. The FE-calculations were carried separately with a non-local modulus of the deformation rate and non-local density factor (stresses, strains and other variables remained local). The error density function (normal Gaussian distribution function) was chosen as a weighting function.

The gradient approach is based on the introduction of a characteristic length by incorporating higher order gradients of strain or state variables into the constitutive law. In the calculations, the first and second gradient of the modulus of the deformation rate or density factor were used. To evaluate the gradient terms, instead of additional shape functions, a standard central difference scheme was employed.

FE-IMPLEMENTATION

The FE-calculations of plane strain compression tests were performed with a sand specimen which was 100 mm high and 20 mm wide. Only quadrilateral finite elements composed of four diagonally crossed triangles were applied to avoid volumetric locking due to dilatancy effects. A quasi-static deformation in sand was initiated through a constant vertical displacement increment prescribed at nodes along the upper edge of the specimen. The boundary conditions of the sand specimen were no shear stress at the top and bottom. To preserve the stability of the specimen against the sliding along the bottom boundary, the node in the middle of the bottom was kept fixed. To numerically obtain a shear zone inside of the specimen, a weaker element with a high initial void ratio was inserted in the middle of the left side of the specimen.

The calculations of a earth pressure problem were performed with a sand body of a height of 200 mm and length of 400 mm. The calculations were carried out with large deformations and curvatures (updated Lagrange formulation). Two sides and the bottom of the sand specimen were assumed to be very rough. The top of the sand specimen was traction and moment free. The retaining wall was assumed to be stiff and very rough. Three different wall modes were assumed in passive and active tests: uniform horizontal translation, rotation around the wall bottom and rotation around the wall top. The calculations were carried out only with a uniform, random and stochastic (with an exponential frequency function) distribution of the initial void ratio.

CONCLUSIONS

The results with a conventional hypoplastic constitutive model suffer from a mesh-dependency. The thickness of shear zones inside of a specimen and load-displacement diagrams are severely mesh-dependent.

A polar, non-local and gradient hypoplastic model provide a full regularization of the boundary value problem during plane strain compression and earth pressure of a retaining wall. Numerical solutions converge to a finite size of the localization zone upon mesh refinement.

The thickness of the localized shear zone and bearing capacity of granular specimens increase with increasing characteristic length.

The distribution of the initial void ratio influences the geometry of shear zones in the granular specimen (passive and active rotation of the retaining wall around the bottom).

The characteristic length within a non-local and gradient theory can be directly related to the mean grain diameter of the granular body.

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