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ON THE USE OF SECOND-ORDER TOPOLOGICAL INFORMATION FOR SUBSURFACE IMAGING BY ELASTIC WAVES

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Summary
This paper is concerned with the generalization of topological derivative, rooted in structural shape optimization, as it pertains to 3D inverse scattering involving elastic-wave identification of subsurface obstacles. Recently, elastodynamic expressions for topological sensitivity have been proposed for the imaging of both semi-infinite and finite solid bodies. Despite their utility, however, these developments are limited in the sense that they are restricted to the identification of subsurface cavities and that they do not provide an explicit link between the nucleated (infinitesimal) cavity and the finite size of a void being sought. To deal with these impediments, the proposed generalization is two-fold and involves: i) development of a formula for the nucleation of a solid inclusion, and ii) rigorous analysis of the second-order topological information that permits direct estimation of the obstacle size through the solution of a canonic least-squares problem, established on the basis of (first-order) topological sensitivity. Numerical examples are included to illustrate the proposed developments.

BACKGROUND

Solutions to inverse scattering problems in elastodynamics, which have a wade range of applications including geophysics, medical imaging, and nondestructive material testing, are derived from either of the three computational platforms that include i) ray-theory approximation [1], ii) finite-difference methods [2], and iii) boundary integral formulations [3]. In the context of three-dimensional subterranean imaging, however, the former two solution methodologies necessitate an extensive computational effort which precludes their full 3D application in all but major land and marine surveys [2]. To mitigate such limitation, boundary-only imaging algorithms have been proposed as a computationally-tractable alternative for solving inverse scattering problems, especially when coupled with the use of gradient-based minimization and analytical shape-sensitivity estimates [4]. Unfortunately, the latter class of solutions necessitate a reliable prior information about the location, topology and geometry of an obstacle for adequate performance.
Motivated by the foregoing considerations, this study deals with the advancement a robust, yet computationally efficient approach for preliminary subsurface imaging by elastic waves based on the concept of topological sensitivity. With reference to a cost functional $\mathcal{J}$ of the body shape, its topological derivative, $\mathcal{T}(\boldsymbol{x}^{\circ})$, synthesizes the first-order sensitivity of $\mathcal{J}$ with respect to the creation of an infinitesimal cavity at a prescribed location $\boldsymbol{x}^{\circ}$ inside the reference, i.e. void-free counterpart of the probed body. The concept of topological derivative first appeared in [5] and [6] in the context of shape optimization of mechanical structures wherein the spatial distribution of $\mathcal{T}(\boldsymbol{x}^{\circ})$ was used as a criterion for the removal of "excess" material through regions where $\mathcal{T}<0$. Recently, elastodynamic generalizations of the topological sensitivity have been proposed in the context of inverse elastic scattering pertaining to semi-infinite [7] and finite [8] domains.

NUCLEATION OF INHOMOGENEITIES

By building on the approach originally proposed in structural shape optimization, developments in [7] and [8] revolve around the process of *cavity* nucleation. As such, their application is limited to inverse scattering problems where subsurface obstacles are in the form of voids. To establish a more general analytical and computational framework that would permit elastic-wave identification of material heterogeneities, the first part of this study deals with the development of an expression $\mathcal{T}(\boldsymbol{x}^{\circ};\mu^*,\nu^*,\rho^*)$ for topological derivative, explicit in terms of the elastodynamic fundamental solution, obtained by an asymptotic expansion of the misfit-type cost functional with respect to the creation of an infinitesimal *inclusion* (with shear modulus μ^*, Poisson's ratio ν^* and mass density ρ^*) in an otherwise homogeneous (reference) solid. Valid for an arbitrary shape of the infinitesimal domain, the formula revolves around the elastostatic solution to six canonical exterior problems, and becomes fully explicit when the vanishing inclusion is spherical. On the basis of the foregoing result, a computational platform is developed for approximate, yet efficient 3D imaging of heterogeneous solids. In contrast to existing solutions which are restricted to the geometric identification of cavities, the proposed approach permits point-wise identification of "optimal" material properties (e.g. μ^*) which minimize the topological sensitivity at a sampling point, $\boldsymbol{x}^{\circ}$. This in turn enables the approach to differentiate between cavities and inclusions, which is a major step toward providing a reliable initial "guess" for more refined, albeit computationally intensive minimization-based imaging algorithms.

SECOND-ORDER TOPOLOGICAL INFORMATION

Numerical experiments show that the topological derivative approach, when applied to inverse scattering, performs best when used in conjunction with wave lengths exceeding the cavity diameter (so-called resonance region). This is perhaps not surprising since the assumption of an *infinitesimal* cavity, implicit to the formulas for topological derivative, is better conformed with by finite cavities that are '*small*' relative to the probing wavelength. Accordingly, the spatial distribution of topological derivative does not furnish direct information about the size of the hidden scatterer, although it may effectively point to its approximate location and shape. As a result, estimation of the obstacle size using first-order topological sensitivity is in practical applications critically dependent on a user-defined threshold parameter. To bridge the gap between the infinitesimal size of a trial "bubble" (inherent to $\mathcal{T}(\boldsymbol{x}^{\text{o}})$) and the finite size of a hidden scatterer, the second part of this investigation is concerned with the rigorous, *second-order* topological expansion of a given cost functional with with respect to obstacle nucleation. It is shown that the explicit knowledge of second-order terms in such elastodynamic expansion, herein obtained by means of a boundary integral analysis, permits direct estimation of the *obstacle size* (whose location is estimated beforehand using the first-order information, $\mathcal{T}(\boldsymbol{x}^{\text{o}})$) through the solution of a canonical least-squares problem that requires only a minimal increase of computational effort. Although herein presented for the case of cavity nucleation only, the proposed second-order analysis is equally applicable to elastic-wave imaging of material heterogeneities by means of the inclusion-type topological sensitivity $\mathcal{T}(\boldsymbol{x}^{\text{o}}; \mu^*, \nu^*, \rho^*)$. A set of numerical examples is included to illustrate the analytical and computational developments.

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