ON INSTABILITY MECHANISMS IN A SEPARATING BOUNDARY-LAYER FLOW

Matthieu Marquillie, Uwe Ehrenstein, François Gallaire
Lab. J.A. Dieudonné, Université de Nice-Sophia Antipolis,
Parc Valrose, F-06108 Nice Cedex 2, France

Summary The stability of separating boundary-layer flow at the rear of a bump mounted on a flat plate is numerically investigated. It is shown that a geometrically controlled, short separation bubble exhibits a global instability consisting of self-sustained two-dimensional saturated perturbations oscillating at a well defined frequency. Local stability analyses confirm that this instability is triggered by a transition from local convective to local absolute instability in the separation bubble. Solving the three-dimensional Navier-Stokes equations, the flow field is shown to exhibit a three-dimensional steady structure well below the critical Reynolds number for the onset of two-dimensional self-sustained oscillations. Regions subject to centrifugal instability due to streamline curvature are detected, considering the Rayleigh discriminant: some evidence is given that the counter-rotating three-dimensional streamwise vortex structure originates from a potentially unstable region nearby the bump-summit.

INTRODUCTION

Separated flow is known to be subject to powerful instability mechanisms. While a backward-facing step is the most canonical geometry to trigger flow separation, and has received a lot of attention during the last decades, in the present investigation smooth bumps mounted on a flat plate are considered, to trigger adverse pressure gradients (which mimic that on the upper side of an airfoil at high angle of attack). Recirculation bubbles are subject to two-dimensional self-sustained oscillations (cf. [1]) and in the present investigation the asssociated global instabilities are analyzed. Recent global stability investigations of the backward-facing step flow have revealed steady three-dimensional perturbations [2], prior (in terms of the Reynolds number) to the onset of global oscillations. Steady three-dimensional disturbances found in the present flow configuration are analyzed with repect to centrifugal instability criteria [3].

RESULTS

The boundary-layer flow evolving along the bump and the adjacent flat plate is numerically computed solving the three-dimensional Navier-Stokes equations using fourth-order finite differences in the streamwise x-direction, Chebyshev-collocation in the wall-normal y-direction and a Fourier expansion in the homogeneous spanwise z-direction, together with a mapping procedure in order to transform the bump-geometry into a Cartesian one.

Synchronized two-dimensional oscillations in controlled separated flow

In a recent investigation [4], numerical simulation of a blowing-suction device in a backward-facing step flow has been shown to be appropriate to produce two-dimensional, globally synchronized oscillatory instabilities. For the present bump-geometry, and solving the two-dimensional Navier-Stokes equations, the elongated recirculation bubble at the rear of a single bump proved to become unstable *via* aperiodic oscillations [5]. Confining however the recirculation bubble using a second bump downstrean the first one which triggers the separation, it is shown that the bubble undergoes indeed global, synchronized oscillations above a critical Reynolds number, which is based in the present investigation on the displacement thickness at inflow where a Blasius boundary-layer profile is imposed (the dimensionless bump height being $h = 2$).

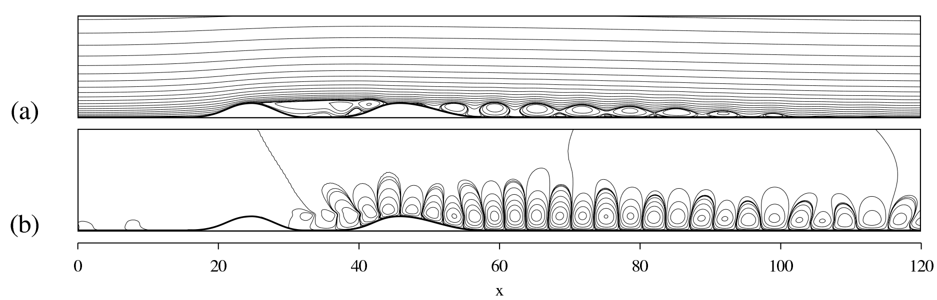
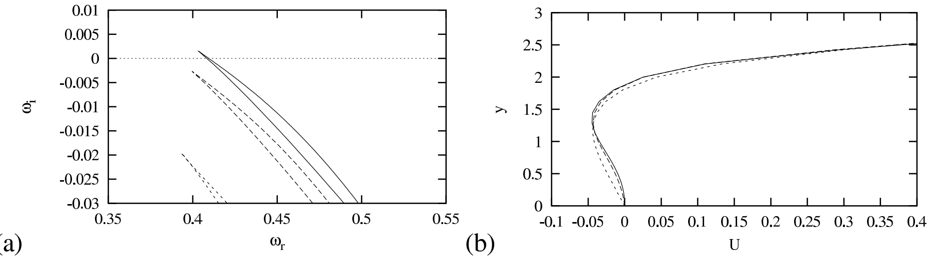


Figure 1: Instantaneous streamlines of (a) the total flow field and (b) the perturbation $\mathbf{u}'$, at $Re = 900$.

The flow field as well as the perturbation are illustrated in figure 1. Local stability analyses, extracting the local velocity profiles from the computed recirculation bubble, have been performed: it is shown that the onset of the global oscillations coincides with the transition from local convective to local absolute instability. As depicted in figure 2(a), the imaginary part of the absolute frequency ω crosses the real axis for increasing Reynolds number, which means that the correponding local profile, shown in figure 2(b), becomes absolutely unstable. The nonlinear frequency-selection criterion for weakly non-parallel flow [6] is shown to predict the computed global frequency.



(a) (b)

Figure 2:(a) Imaginary part ω_i of complex frequency at the cusp $\frac{\partial\omega}{\partial\alpha}(\alpha_0) = 0$ as function of ω_r and (b) the corresponding profiles, at $x = 33$; ...: $Re = 600$, - - -: $Re = 850$,—: $Re = 900$.

Three-dimensional steady state

Advancing in time solving the three-dimensional Navier-Stokes equations, the flow field converged as expected to a two-dimensional steady state for low inflow Reynolds numbers. However, increasing the Reynolds number, three-dimensional flow structures appear with growing (nonlinear) disturbance amplitudes, well before two-dimensional oscillations set in, similar to backward-facing step flow [2]. Three-dimensional steady disturbances may occur in two-dimensional flow structures which exhibit local vorticity of opposite sign than that of the local algebraic radius of curvature of the streamlines: indeed, classically the Rayleigh dicriminant $R(x, y) = 2U\omega_z/r$ to be negative provides a necessary condition for the corresponding centrifugal instability, where U is the modulus of velocity, ω_z the vorticity of the 2D flow-field (u, v) and r the local radius of the streamlines. Isolines for negative values of the Rayleigh discriminant are depicted in figure 3, for a two-dimensional steady state at $Re = 530$. The upstream part of the bump, where the latter one has concave curvature, exhibits an important region of negative R-values and a second region unstable in Rayleigh's sense appears nearby the summit of the bump. Multiple regions of negative Rayleigh discriminants are also reported in [3] for backward-facing step flow. Figure 4 depicts the three-dimensional flow structure downstream the bump, isosurfaces of the streamwise vorticity being shown. The region of negative R-values is superimposed as well and interestingly the counter-rotating vortex structure is seen to be attached to the potentially unstable region at the summit of the bump.

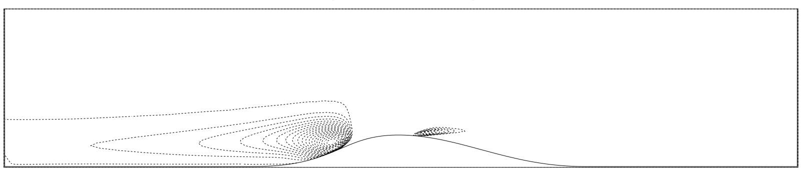


Figure 3: Isolines of negative $R(x, y)$-values indicating regions of possible centrifugal instability.

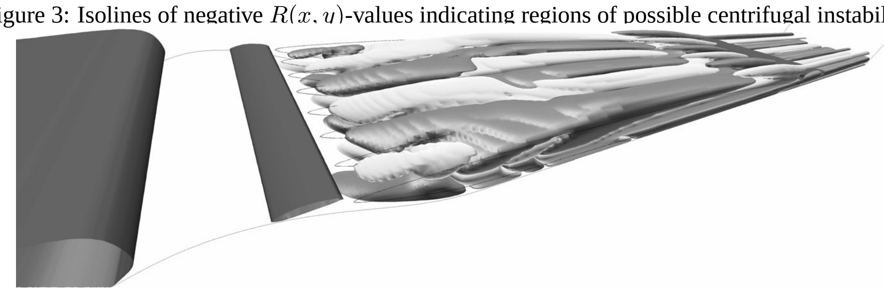


Figure 4: Isosurfaces of $\omega_x = \pm$const.; grey : $\omega_x > 0$; white: $\omega_x < 0$. Black: regions subject to centrifugal instability with $R < 0$.

DISCUSSION

Two different instability mechanisms are identified: while two-dimensional oscillations may clearly be attributed, for confined recirculation bubbles, to local transition from convective to absolute instability, it is conjectured that the global three-dimensional steady states are related to local curvature effects. While the Görtler-type instability mechanism at the upstream part is shown to be inoperative, flow acceleration at the convex part of the bump possibly annihilates the development of perturbations, counter-rotating streamwise vortices originate at the summit of the bump, at the very region where the two-dimensional flow topology satisfies the Rayleigh criterion for (inviscid) centrifugal instability.

References

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