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HOMOGENIZATION OF PLAIN WEAVE COMPOSITES WITH IMPERFECT MICROSTRUCTURE

Jan Zeman*, Michal Šejnoha*
*Department of Structural Mechanics, Faculty of Civil Engineering, Czech Technical University in Prague, Thákurova 7, 166 29, Prague, Czech Republic

Summary A comprehensive computational framework is presented for the determination of a representative volume element (RVE) of plain weave fabric composites with reinforcement imperfections. The presented approach is characterized by quantification of the analyzed material system by appropriate statistical descriptors that are subsequently used to derive the parameters on an idealized geometrical model by minimizing the difference between the original and the idealized unit cell.

INTRODUCTION

The doubtless benefits offered by composite materials such as a high strength, light weight, corrosion resistance and design flexibility resulted in an extensive use of these materials in diverse applications in various areas of engineering. In particular, the popularity of the woven fabric reinforced composites is under continuous rise due to advantageous strength/weight ratio, easiness of manipulation and low production costs.

The complex three-dimensional structure of woven fabric composites, however, makes the analysis and prediction of the overall properties of these material systems a relatively difficult task. Moreover, it is an experimentally confirmed fact that an overall response of such structures is highly influenced by both the material behavior and *geometrical imperfections* of distinct phases of the composite system. A comprehensive discussion of this subject can be found, e.g., in [1] where detailed image analysis of composite micrographs was used to get the frequency spectrum to describe the yarn shape of both woven and braided fabrics. Moreover, it has been recognized that the crimp wavefront is subject to deformation when pressed during the manufacturing process and that the frequency spectrum is therefore disordered compared to that of the free fabric.

In the present contribution, an alternative strategy is adopted to incorporate, at least to some extent, the reinforcement imperfections into the geometrical model of the unit cell. In particular, we follow the path set in papers by [4] and by [6]. In these works, the idealized geometrical model of the analyzed composite is defined in terms of a certain periodic unit cell with geometrical parameters derived by matching microstructural statistics of a real microstructure and the searched PUC. Using the model of plain weave geometry [2] in combination with binary images of real composites, this modeling strategy can be extended to the modeling of woven composites in a rather straightforward way.

MICROSTRUCTURE DESCRIPTION

In the present study, two different statistical descriptors – the two-point probability function S_{rs} and the lineal path function L_r are used to quantify the morphology of random media. Recall that the two-point probability function $S_{rs}(\mathbf{x})$ gives the probability that both endpoints of the vector $\mathbf{x}$, randomly thrown into material, will be located in phases r and s[1], respectively. The lineal path function $L_r(\mathbf{x})$ simply gives the probability that the segment $\mathbf{x}$ will be fully contained in the phase r. Note that the two-point probability function was evaluated by the Fast Fourier transform-based algorithm while the sampling template approach was used for the lineal path functions; see recent monograph [5] for more details.

GEOMETRICAL MODEL

In this work, the model of fabric weave composite proposed in [2] is used since it is reasonably simple to implement and directly incorporates typical features of real composites. For the purpose of the present study, the most important fact is that the the model is fully determined by four parameters a, b, g and h, see Fig. 1; detailed description of the model geometry can be found in [2].

OPTIMIZATION PROBLEM

To determine "statistically" optimal parameters of the periodic unit cell, the parameters a, b, g and h are found by minimizing certain objective functions. In particular, objective functions incorporating the two-point matrix probability function S_{mm}, matrix lineal path function L_m or their combination are considered,

$$F_S(\mathbf{x}) = \sum_{i=-i_{\max}}^{i_{\max}} \sum_{j=-j_{\max}}^{j_{\max}} \left(\overline{S}_{mm}(i,j) - S_{mm}(i,j)\right)^2, \qquad F_L(\mathbf{x}) = \sum_{i=0}^{N_d-1} \sum_{j=0}^{N_\ell(i)-1} \left(\overline{L}_m(i,j) - L_m(i,j)\right)^2 \quad (1)$$

[1]Note that a binary medium is considered in the text with symbol m standing for matrix and f denoting the bundle; i.e., $r \in \{m, f\}$.

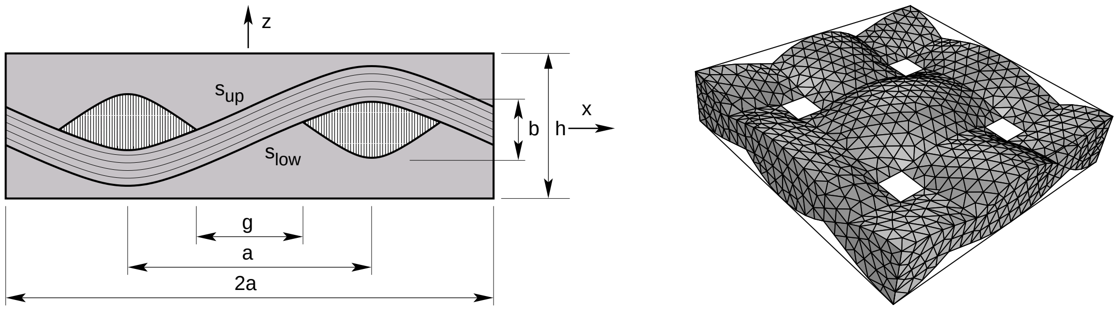

Figure 1. Idealized Periodic Unit Cell, (a) Geometrical model, (b) Example of finite element mesh

where $\overline{S}_{mm}$ and $\overline{L}_m$ are the values of L_m and S_{mm} functions corresponding to the target microstructure. A closer inspection reveals that the objective functions (1) are discontinuous with a large number of local plateaus. This is a direct consequence of working with binary images of a limited resolution. Based on our previous works [3], the *Real-encoded Augmented Simulated Annealing* method is employed to solve the present problem.

NUMERICAL EXPERIMENTS

As a representative of digitized images of real-world multilayered plain weave composites, a set of three artificial bitmaps exhibiting different imperfections was generated. In particular, the "samples" formed by two unit cells with different layer and bundle heights (see Fig. 2a), two identical unit cells shifted by a (see Fig. 2b) and by $a/2$, Fig. 2c were considered.

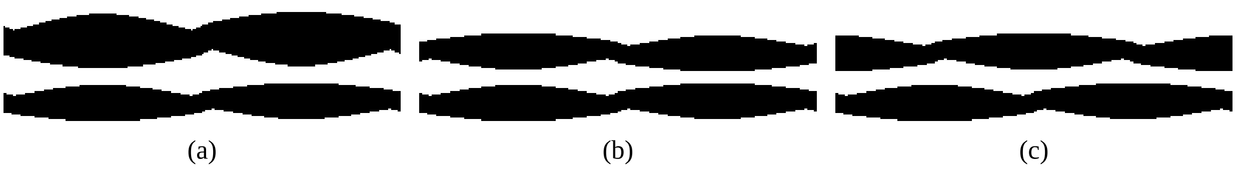

Figure 2. Artificial bitmaps of mesoscale geometry with typical tow misalignments, (a) different layer heights (PUC #1), (b) layers shifted by a (PUC #2), (c) layers shifted by $a/2$ (PUC #3)

Finally, we present the comparison of effective elastic properties for the target bitmaps and corresponding statistically optimized unit cells obtained for different objective functions. The material parameters of the matrix phase and the bundle were taken from [6].

PUC #1				***PUC #2***				***PUC #3***			
S_{mm}	L_m	$S+L$	Target	S_{mm}	L_m	$S+L$	Target	S_{mm}	L_m	$S+L$	Target
21.100	22.698	23.334	23.324	25.341	26.309	25.121	24.786	27.629	27.464	27.095	24.694

Table 1. Coefficient $\mathsf{L}^{\mathsf{fem}}{}_{11}$ of the homogenized stiffness matrix [GPa].

The obtained results allow us to conclude that the proposed procedure can be efficiently used for multilayered composites with possibly varying layer heights (PUC #1) provided that the relative shift of individual layers is not very large or approximately equal to the unit cell half-width (PUC #2). In the opposite case, however, it appears to be necessary to formulate the optimized unit cell in terms of at least two-layered composite.

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