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A DYNAMIC UNILATERAL CONTACT PROBLEM FOR A CRACKED BODY

Marius Cocou*, Eliane Bécache**, Patrick Joly**, Gilles Scarella**
*Laboratoire de Mécanique et d'Acoustique CNRS, 31, chemin Joseph Aiguier, 13402 Marseille Cedex 20, and Université de Provence, Marseille, France
**INRIA Rocquencourt, Domaine de Voluceau, B.P. 105, F-78153 Le Chesnay Cedex, France

Summary In this work we investigate a class of dynamic contact problems for cracked viscoelastic and elastic bodies, when Signorini's conditions between the two faces of the crack are considered. Firstly, using a penalty method we study a variational formulation of a unilateral contact problem with nonlocal friction for a cracked viscoelastic body. Several estimates on the penalized solutions are obtained which enable us to analyze the time and spatial discretizations of the problem. Then we consider the corresponding elastic problem, for which a fictitious domain formulation is proposed with Lagrange multipliers representing the normal jump of the displacements. Numerical examples, based on the fictitious domain method for solving the diffraction of elastic waves by cracks, are presented.

INTRODUCTION

In this paper we study some dynamic unilateral contact problems with friction for a cracked (visco)elastic body, see [5], [1], [10] and the references therein.
For the wave equation with unilateral boundary conditions, theoretical results were proved in the case of a half-space by G. Lebeau and M. Schatzman [8], in the case of a smooth bounded domain by J.U. Kim [7] and for a thermoelastic radially symmetric body by J. Muñoz-Rivera and R. Racke [9]. For a viscoelastic behaviour, J. Jarušek [6] was the first to obtain an existence result for the dynamic unilateral contact problem with given friction. In [4] dynamic contact problems with nonlocal friction were studied under boundedness hypotheses on velocity and acceleration and in [3] an existence result was proved for a dynamic Signorini's problem with nonlocal friction, without any additional assumptions.

ANALYSIS OF A VISCOELASTIC PROBLEM

In this section, using a penalty method we study the primal variational formulation of a unilateral contact problem with nonlocal friction for a cracked viscoelastic body of Kelvin-Voigt type.
We formulate a penalized contact problem with the solution verifying the same equations in the bulk and the same boundary conditions as in the initial problem, except for unilateral contact conditions. The penetration between the two faces of the crack is penalized.
An equivalent formulation using a decomposition of the domain is presented and time discretizations of the penalized problem are analyzed. Several estimates are obtained on the penalized solutions, which enable us to pass to the limit by using compactness results.

FICTITIOUS DOMAIN FORMULATION

We consider the corresponding elastic problem, for which a fictitious domain formulation is proposed with Lagrange multipliers representing the normal jump of the displacements, see [1].
Following a similar approach as in [2], where a free surface boundary condition was studied, we use a fictitious domain method for solving this problem. The main advantage of this method is to offer the efficiency of the finite difference method while taking into account the complex geometry of the crack, through the Lagrange multipliers defined on the crack. By this method the corresponding space discretization requires two meshes: a regular one of the bulk and another one of the crack.

NUMERICAL RESULTS

Numerical results are based on the fictitious domain method for solving the diffraction of elastic waves by cracks.
For example, in the case of radial symmetry for a circular crack unilateral contact condition is compared to free surface condition, for the same initial conditions and given forces, at the same time (see Figs. 1,2).
A second example concerns a vertical crack for which we compare, at the same time and for the same given conditions, unilateral contact solution to free surface solution (see Figs. 3,4).

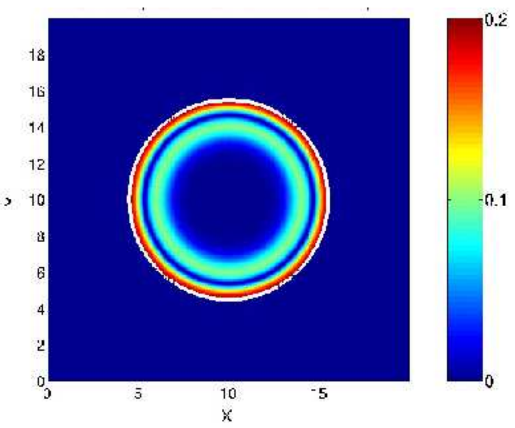

Figure 1. Circular crack, free surface. Norm of the displacement.

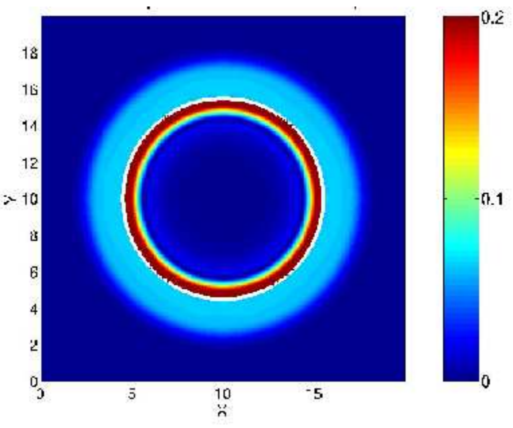

Figure 2. Circular crack, unilateral contact. Norm of the displacement.

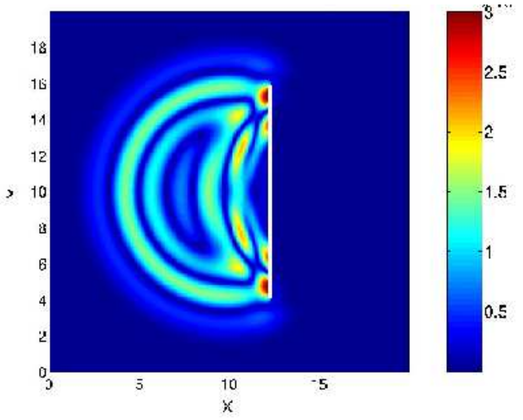

Figure 3. Vertical crack, free surface. Norm of the displacement.

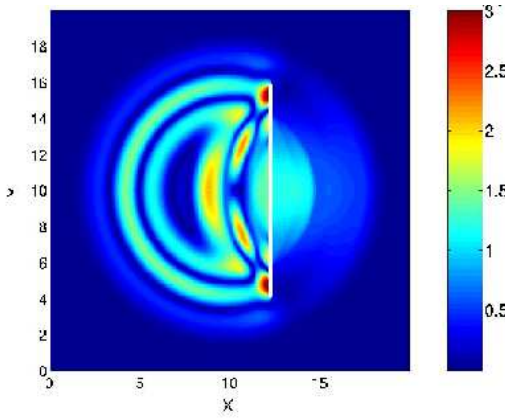

Figure 4. Vertical crack, unilateral contact. Norm of the displacement.

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