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STATIC SHAPES OF LEVITATING VISCOUS DROPS

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Summary A lubrication model for the levitation of a drop of molten glass above a spherical porous mould is described, and used to find static shapes of the drop. Comparison with full Navier-Stokes simulations is excellent, and a condition is deduced for the stability of these solutions.

INTRODUCTION

We consider the levitation of a drop of molten glass above a spherical porous mould, through which air is injected with a constant velocity v. In the present context, we assume that the glass is so viscous compared to the air that we can neglect the motion in the drop. Therefore, if static shapes of the drop exist, these shapes are completely determined by the coupling between the equations of motion in the air cushion and the Young-Laplace equation. In what follows, all the lengths are made dimensionless with the capillary length $\sqrt{\sigma/\rho g}$, where σ is the surface tension coefficient of a glass-air interface, ρ is the density of the glass, and g the acceleration due to gravity.

LUBRICATION MODEL

The sessile top surface

The computation of these static shapes can be split into two parts. The first consists in solving the equations for the upper surface, on which the pressure is atmospheric. Taking account of the surface tension, the equation governing this sessile solution is

$$\kappa(r) - z(r) = C^{ste} = \kappa(0), \tag{1}$$

where r is the distance to the axis of symmetry and the curvature $\kappa(r)$ can be approximated by the full 2D curvature

$$\kappa(r) = \frac{z''(r)}{[1 + z'^2(r)]^{3/2}}, \tag{2}$$

owing to the large aspect ratio of a flat-lying drop. This equation is subject to the boundary conditions

$$z'(0) = 0 \qquad \kappa(0) = \kappa_0.$$

Introducing the arclength s and the angle φ between the normal vector to the interface and the vertical, we can split equation (1) into three first-order ordinary differential equations for $\varphi(s)$, $r(s)$ and $z(s)$. These equations are integrated using a fifth-order adaptative Runge-Kutta method, from the top of the drop to a point $r = R$ near where the drop approaches the mould.

The lubrication problem

Once the top surface is known, we match this solution to a lubrication solution in the air film, starting at $r = R$. Let v denote the inlet velocity through the porous mould and μ the dynamic viscosity of the air. Assuming the mould to be almost flat, the lubrication approximation links the depth integrated flux Q at a distance r from the axis of symmetry to the pressure gradient in the r direction [1] :

$$Q = -\frac{h^3}{12\mu}\frac{\partial p}{\partial r}$$

By mass conservation, $Q = rv/2$. In this study, we assume that the motion in the glass can be neglected, so that the non-dimensional pressure in the gas (independent of z, as a consequence of lubrication theory) is

$$p(r) = p_0 - (f(r) + h(r)) - \kappa(r),$$

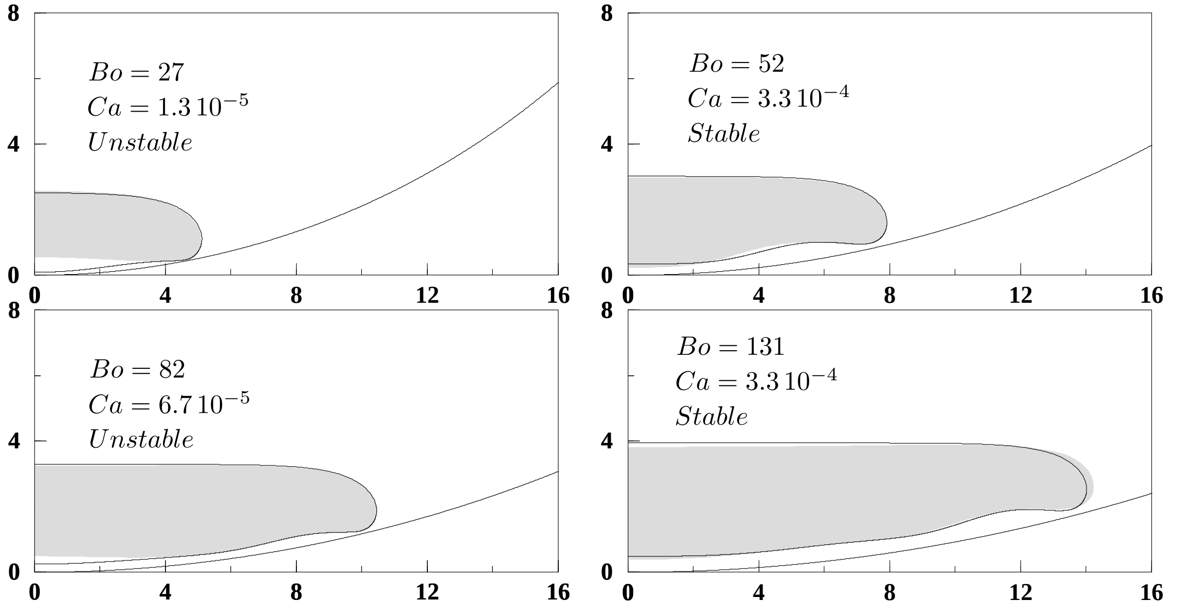
where $f(r) + h(r)$ is the hydrostatic pressure in the glass, $h(r)$ is the mould shape and the curvature is again taken to be the full 2D curvature (2). Therefore, the non-dimensional steady lubrication equation is

$$6Ca = \frac{h^3}{r}\left(f + h + \frac{f'' + h''}{[1 + (f' + h')^2]^{3/2}}\right)', \tag{3}$$

where $Ca = \mu v/\sigma$ is the capillary number. Once again, we split equation (3) into three first-order ordinary differential equations for $h(r)$, $h'(r)$ and $h''(r)$, where $h'(R)$ and $h''(R)$ are given by the sessile solution and $h(R)$ is a shooting parameter chosen such that the slope on the axis of symmetry is equal to zero.

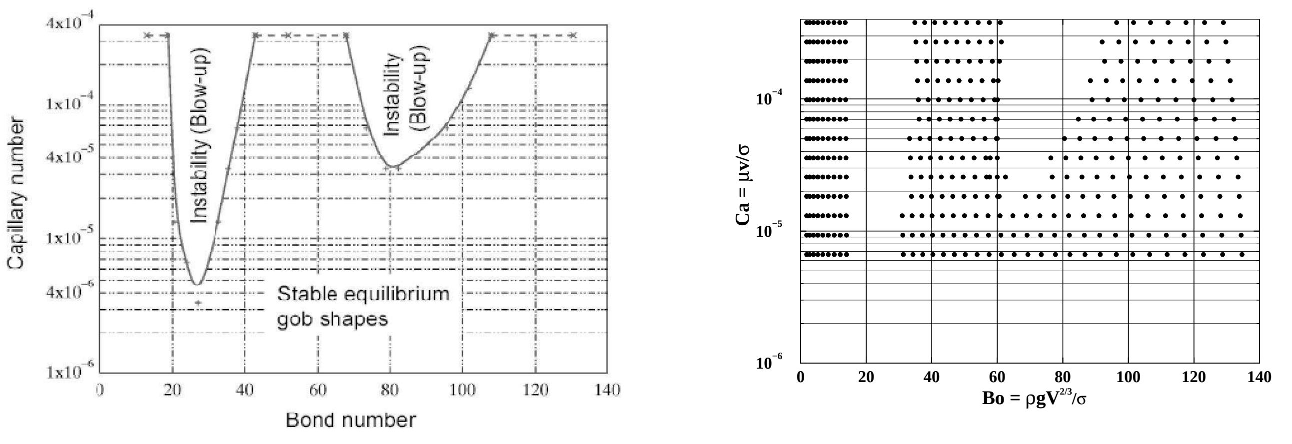
COMPARISON WITH NAVIER-STOKES SIMULATIONS

Some simulations of the full set of Navier-Stokes equations have been made with an industrial code (POLYFLOW) [2]. Figure 1 shows the comparison between four of these simulations and our lubrication model. The gray patches correspond to the POLYFLOW computations, and the solid lines to the lubrication model. In this example, the radius of curvature of the spherical mould R_M is equal to $4.76V^{1/3}$, where V is the volume of the drop. Some of the Navier-Stokes simulations



$Figure\,1.$

are stable (the two on the right), and some are unstable (the two on the left). Figure 2 shows, on the left, the stability diagram corresponding to the POLYFLOW simulations [2], according to the Bond number $\rho g V^{2/3}/\sigma$ and the capillary number $\mu v/\sigma$. On the right of the graph are shown some of the lubrication results, for a constant radius of the mould equal to 50 : the points correspond to simulations for which the thickness of the thin film never reaches 1 (which is the capillary length). These results show clearly two regions in which there isn't any realistic solution. This may be a starting point for understanding the stability.



$Figure\,2.$

References

[1] Hinch E.J., Lemaitre J.: The effect of viscosity on the height of disks floating above an air table. *J. Fluid Mech* **273**:313–322, 1994.

[2] Lange, U.: Gas-Film Levitation of Viscous Glass Droplets, ECMI Glass Days, Wattens, Austria, 21/11/2002