Plane Couette flow of dense liquid-particle suspensions

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Dense solid-liquid suspensions are encountered in numerous situations in nature (e.g. sediment transport in rivers and oceans) and in industry (e.g. fluidized beds, slurry reactors). The specific flow problem that we study is inspired by the classical experimental study of Bagnold [1], who employed a cylindrical Couette device to measure shear and normal stresses in neutrally buoyant suspensions as functions of apparent shear rate and volume fraction of solids. He concluded from his experiments that at high shear rates both shear and normal stresses manifested a quadratic dependence on apparent shear rate, labeled this as a grain-inertia regime, and attributed the stresses in this regime to particle-particle interactions. Recently, Hunt et al. [2] critically assessed the design of Bagnold's experiments and the single-phase flow that would occur in this geometry, and raised troubling uncertainties about his interpretation. To address some of the issues raised by Hunt et al. [2], and to study the role of solid particle motion, we have simulated the behavior of dense (solids volume fractions of 25-60%) solid-liquid suspensions undergoing simple shear.

Our simulation procedure, which is three-dimensional and time-dependent, applies lattice-Boltzmann discretization of the Navier-Stokes equations to solve the fluid flow in between the particles [3]. The solid particles are represented by an immersed boundary technique [4]. This method directly provides the hydrodynamic forces acting on the particles. Particles are allowed to interact through hard-sphere, inelastic, frictional collisions. We include lubrication forces between solid surfaces, when the gap between the surfaces is smaller than the lattice spacing [5].

The flow domain in our simulations is bounded by two flat walls, which create a shearing motion by moving in opposite directions. At the remaining four bounding planes, periodic boundary conditions are imposed. Just as in Bagnold's experiments, the gap width amounts to 8 particle diameters. In the examples shown below, the lengths of the domain in streamwise and spanwise directions are 16 and 8 particle diameters respectively. We have performed simulations at different resolutions to ascertain that a particle diameter of 16 lattice spacing is adequate to get essentially grid-spacing independent results. At this resolution, the size of the flow domain is 256x128x128 lattice units. From our simulations, we determine how the time-averaged particle concentration, fluid and particle velocities, fluid-particle interaction force, shear and normal stresses due to particle-particle interactions, fluid phase viscous and pseudo-turbulent stresses vary across the gap.

In Figure 1, we present some of our results in a qualitative manner. At particle volume fractions of 0.50 (Figure 1D) and 0.55 (Figure 1E), the shear orders particles in strings along the flow direction (shear induced crystallization). This effect has been observed in colloidal systems, where the Stokes number is very small [6]. In the cases presented in Figure 1, however, the Stokes number (defined as $St=\frac{\rho_s}{\rho_l}\frac{\dot{\gamma}d_p^2}{\nu}$ where $\dot{\gamma}$ is the apparent shear rate, d_p is the particle diameter, ν is the fluid's viscosity; and the density ratio is taken to be 1 to obtain a neutrally buoyant system) amounts to 375. At 59% solids (Figure 1F) the ordering is much less pronounced. The ordering largely disappears when we introduce a little polydispersity (Figure 1G). At relatively low solids volume fractions (see Figure 1A), particles tend to avoid wall regions and hardly ever collide with the walls.

Quantitative analysis of the 46% solids cases (see Figure 2) reveals the presence of a granular regime (with the dimensionless normal stress approximately proportional to Ba^2). No evidence for the macro-viscous regime in which the stress is proportional to Ba was found even at Ba as small as 10. Further

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analysis showed that in most of the flow domain momentum is mainly transferred by particle-particle collisions, and that lubrication forces play a minor role in this process.

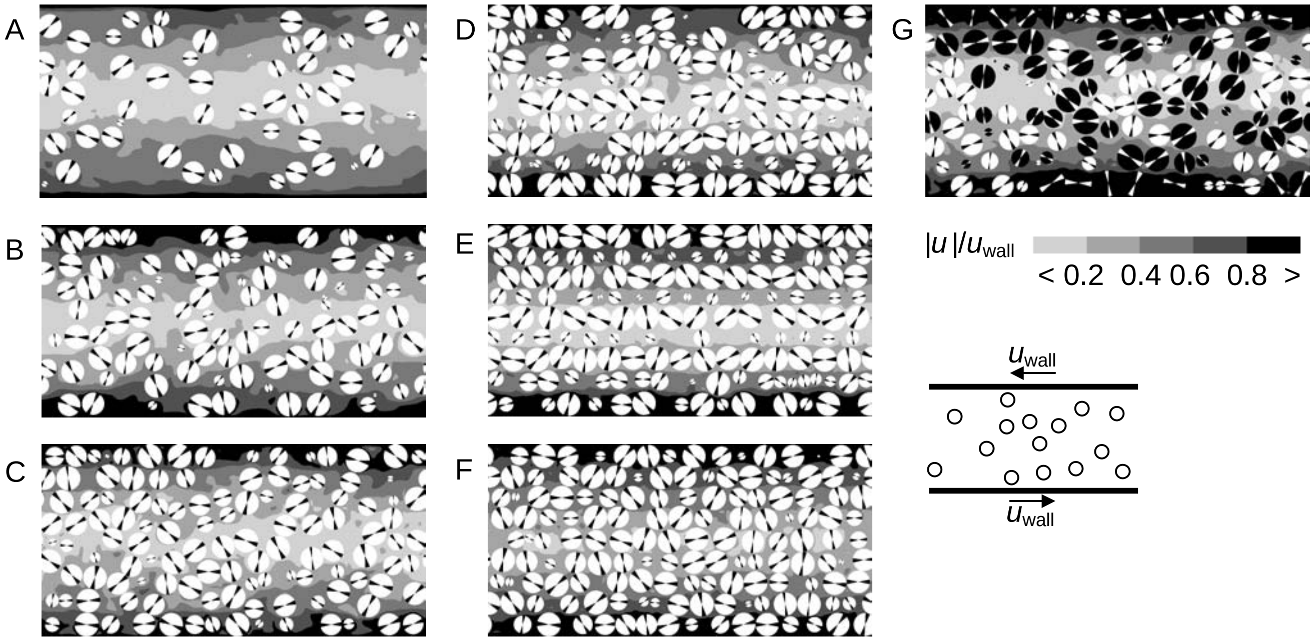

Figure 1. Cross-sections of single realizations of the solid-liquid flow. In cases A-F the particles have uniform size and *St*=375 (*St* is defined in the text). The solids volume fractions in cases A-F are 0.26, 0.37, 0.46, 0.50, 0.55, and 0.59 respectively. Case G has the same solids volume fraction as case D, except that in case G there are two particle diameters: one 0.9 times (white), and one 1.1 times (black) the diameter of case D. The gray scale denotes the absolute value of the liquid velocity. The collisions are elastic and frictionless.

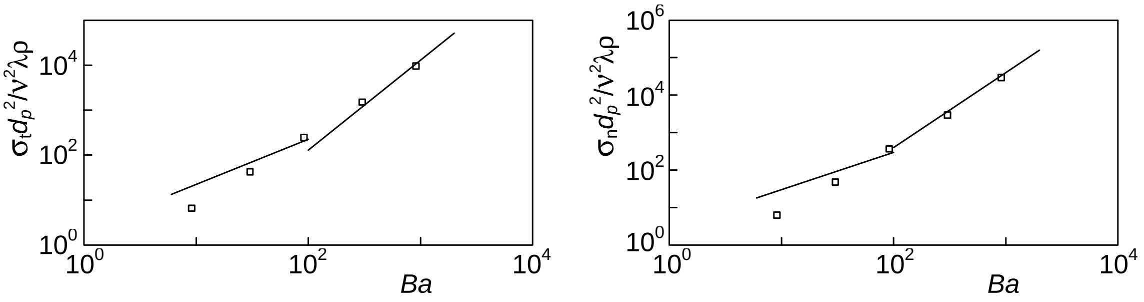

Figure 2. Non-dimensional shear (left) and normal (right) stress as a function of the Bagnold number ($Ba=St\cdot\lambda^{½}$ if $\rho_s=\rho_l$, with $\lambda=1/[(\phi_0/\phi)^{1/3}-1]$, and $\phi_0=0.74$ the closest packing of spheres volume fraction). The solid lines are the empirical correlations due to Bagnold [1], the symbols are simulation results for $\phi=0.46$ ($\lambda=5.8$).

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