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THERMOELASTIC RELAXATION IN THIN PLATES WITH APPLICATIONS TO MEMS AND NEMS OSCILLATORS

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Summary Equations governing thermoelastic loss in vibrating thin plates are derived, generalizing Zener's theory of thermoelastic loss in beams. The mechanism for the loss of mechanical energy is thermal diffusion caused by inhomogeneous deformation, flexure in thin plates. It is shown that a critical plate thickness, h^*, exists which separates two distinct types of response, which may be called thermally thick and thin. For plates of thickness $h > h^*$ the diffusion, and hence the thermal component of the solution, is restricted to a 1-dimensional heat flux across the plate thickness. If the thickness is less than h^* then in-plane thermal diffusion cannot be ignored, and may in fact dominate. Several general results are obtained for plates with $h > h^*$. Thus, using the Kirchhoff assumption for the elastic deformation, it is shown that the local thermal relaxation loss depends upon the local state of vibrating flexure, specifically, the principal curvatures at a given point on the plate. The thermal loss is zero at points where the principal curvatures are equal and opposite, i.e. saddle shaped deformation. Conversely, thermal loss is maximum at points where the curvatures are equal, or spherical flexure. An effective plate equation is derived that incorporates the thermoelastic loss as a damping term. The general form of the effective damping that is obtained for $h > h^*$ can be generalized to arbitrary thickness, in particular the case of thermally thin plates can be considered. These results are useful in predicting mode widths in MEMS and NEMS oscillators.

INTRODUCTION

High Q resonators are central to the development of new devices and applications that include RF filters, next generation MRI systems, and torque magnetometers. Si-based MEMS/NEMS oscillators are the candidates of choice, and include free standing planar devices, such as double paddle oscillators (DPOs), and micro-cantilevers. As the oscillators shrink in size, it has been found that the Q achieved is orders of magnitude smaller than expected based on classical fundamental loss mechanisms. Many mechanisms have been proposed, including surface loss that increases with the surface to volume ratio. However, under controlled conditions with minimal surface defects and adsorbates, measurements on Silicon DPOs have shown that room temperature losses are adequately described by thermoelastic relaxation, while unexplained mechanisms operate at lower temperatures. Interestingly, the mode of vibration of DPOs is designed to be primarily torsional with very little flexure (and hence no thermoelastic coupling). However, as demonstrated by Photiadis et al. [1] it is precisely the small amount of flexural motion that accounts for loss in these supposedly torsional oscillators. In this talk we describe a consistent theory for predicting intrinsic dissipation arising from thermoelasticity in elastic structures. Particular attention will be given to flexural motion of thin plates. This work is a step towards understanding the fundamental limits of dissipation in small structures such as NEM and MEM devices.

Surprisingly few papers have appeared on the topic of damping of structural vibration via thermal relaxation since Zener published his now-classic theory of anelastic thermoelastic damping, see [2]. Alblas [3] provided a rigorous formulation using continuum thermomechanics and linear elasticity theory for isotropic materials. He derived detailed and explicit solutions for the thermoelastic damping in vibrating beams, including the circular rod and the rectangular beam. The result for the latter was derived separately by Lifshitz and Roukes [4], although Alblas' solution is the more general of the two. Perhaps the most thorough analysis of thermal damping in the context of the coupled equations of thermoelasticity is due to Chadwick [5]. By considering a modal decomposition of the elastic and thermal fields, an exact relation for the complex valued frequency of oscillation of each mode was obtained. This enabled Chadwick to derive a generalization of Zener's expression for the thermoelastic damping of an arbitrary elastic body, and in a far more rigorous way than the original. Chadwick subsequently derived the governing equations of thermoelasticity for thin plates and beams [6]. The equations are in the form of coupled equations, one of which reduces to the classical equations for the structural mode, e.g. flexural waves in thin plates, and the other the temperature diffusion equation, in the limit of zero coupling. The analysis in [6] is restricted to isotropic solids, and no specific structures are considered, only general bodies for which the solution is expressed in terms of the eigenfunctions of Laplace's equation.

THEORY

We are concerned with determining the thermomechanical loss of elastic modes, e.g. the flexural mode of a rectangular thin plate. The key to this approach is the valid assumption (implicit in Zener's work) that there is little relative difference between the isentropic (unrelaxed) and isothermal (relaxed) mechanical responses, and hence the mechanical and thermal problems are essentially decoupled. Since the thermoelasticity is weak, the transition from the instantaneous or unrelaxed system to the relaxed system can be viewed as a quasistatic thermal process, governed by the normal equations for thermal diffusion, although now in the presence of an inhomogeneous deformation.

We employ the general equations of coupled thermoelasticity for anisotropic materials. In addition, we assume the standard form for the heat flux, which makes the dynamic system irreversible and dissipative. The analysis takes advantage of the universally small coupling between the thermal and elastic fields, permitting an asymptotically exact formulation in the small parameter $\epsilon = E\alpha^2 T/C_p$[1], see Table I. Using the classical Kirchhoff form for the plate deformation, the governing equations are adapted to thin plate configurations. This yields equations for the moments and stresses in terms of the usual elastic deformation plus certain moments of the temperature field θ. The key to obtaining explicit plate equations independent of temperature is to solve the temperature equation, which is in the form of a forced thermal diffusion problem. The forcing depends upon the stress field in the plate, and is greatest in regions of hydrostatic stress, which implies the most significant effects are to be expected from flexural waves.

RESULTS

In deriving effective plate equations for flexural wave motion, we find that there is a characteristic length, h^*(see Table I). This arises from the fact that both the flexural wavenumber and the thermal diffusion wavenumber are proportional to $\omega^{1/2}$, and there is a unique plate thickness when they are commensurate. For plates thicker than this length the thermal diffusion problem can be solved and explicit plate equations found. For instance,

Material	100 ϵ	h^* (μm)
Gold	0.017	5,685
Silicon	0.022	272
SiC	0.059	43
Platinum	0.13	534
Silver	0.22	8028
Nickel	0.27	239
Copper	0.28	2681
GaAs	0.48	16
Beryllium	0.56	141
Lead	0.85	5739
Aluminum	4.7	53

Table I. The thermoelastic coupling parameter ϵ at 300 K, and the length h^* for different materials.

isotropic materials reproduce the classical plate equation $D\nabla^4 u - \rho h\omega^2\, u = 0$ where now the bending stiffness is

$$D = \frac{EI}{1-\nu^2}\big(1+\epsilon f(kh)\big), \qquad f(\zeta) = \frac{24}{\zeta^3}\left[\frac{\zeta}{2} - \tan\frac{\zeta}{2}\right],$$

where h is the thickness, and k is the complex thermal wavenumber. The dissipation depends on the imaginary part of D. Generalizations to orthotropic plates will be presented.

A general result governing the rate of *local* energy loss in thin plates is also derived. Specifically, it is shown that the relative decay of flexural energy at a point on a plate is of the form (for isotropic plates)

$$\eta = \epsilon\, g(\omega)\, \frac{\left(\kappa_1+\kappa_2\right)^2}{\left(\kappa_1+\kappa_2\right)^2 - 2(1-\nu)\kappa_1\kappa_2}$$

where κ_1 and κ_2 are the two principle curvatures at the point, and the function g depends only on frequency and thickness. Thus, thermoelastic damping vanishes at oscillating saddle points, and is maximum for locally "spherical" bending.

For plates of thickness less than h^* the in-plane thermal diffusion can be taken into account, although effective plate equations cannot be obtain. We can however derive dispersion relation for flexural waves, which depend upon the same function f above although in an implicit manner. This permits us to extend the present theory to NEMS rather than just MEMS.

References

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[6] Chadwick, P.: On the Propagation of Thermoelastic Disturbances in Thin Plates and Rods. *J. Mech. Phys. Solids* **10**:99–109, 1962.

[1] E, α, T and C_p are respectively Young's modulus, thermal expansion coefficient, absolute temperature and heat capacity