FOCUSING OF AN INERTIA-GRAVITY WAVE PACKET BY A BAROCLINIC SHEAR FLOW

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Summary
We investigate the interaction of an internal gravity wave packet in a rotating fluid with a baroclinic shear flow, using ray equations and three-dimensional direct numerical simulations (DNS) of the Boussinesq equations. In this problem, the intrinsic frequency of the wave packet increases due to the horizontal shear of the background flow in which it propagates. The packet is trapped at locations where the intrinsic frequency reaches its upper bound and is amplified there. When the horizontal shear of the background flow is low enough, ray equations predict that the packet may further penetrate into that flow through reflection within a wave guide. The DNS show that the packet is actually dissipated before reflecting because of the strong decrease of its horizontal wavelength and group velocity during the interaction. Consequently, the wave packet is not able to penetrate into the shear flow, except when the latter vanishes locally.

INTRODUCTION

The purpose of this paper is to investigate how inertia-gravity waves, *i.e.* internal gravity waves in a rotating medium, may interact with a horizontal flow with both a horizontal and a vertical shear. Such interactions occur everywhere in geophysical flows as soon as inertia-gravity waves propagate in a wind or a current, or encounter a vortical motion such as a large scale vortex. We use a simple model for the wave and the baroclinic flow, described in the next section. We rely on ray theory and also perform direct numerical simulations of the three-dimensional nonlinear Boussinesq equations. The geophysical motivation of this work is to investigate whether some irreversible transport process may occur across the shear flow as a result of its interaction with the wave field.

PHYSICAL MODEL

Let (x, y, z), with z directed upwards, be a Cartesian coordinate system in the rotating reference frame attached to the fluid container. The baroclinic shear flow consists of a velocity field along the x-direction $\mathbf{U}(y, z)$ in thermal wind balance with a buoyancy field $B(y, z)$. The baroclinic current is a horizontal shear layer, centered about $y = y_s$, with a vertical shear: $U(y,z)/U_0 = \left[1 + \tanh\left(\frac{y-y_s}{L_s}\right)\right]\left[1 + \beta \sin\left(\frac{2\pi z}{H_s}\right)\right] - 1$. The velocity scale is U_0 and the parameter β represents the strength of the baroclinicity: the shear flow is barotropic if $\beta = 0$ or $\beta \ll 1$. The buoyancy field B is inferred from the thermal wind balance. This initial condition implies that an inertia-gravity wave packet propagating from a region where $y \ll y_s$ travels, as y increases, from a uniformly translating medium along the x direction with speed $-U_0$, to a moving medium with both a vertical and a horizontal velocity shear. Let $\mathbf{k}$ and $\Omega(\mathbf{k})$ refer to the main wave vector and intrinsic frequency respectively of such a wave packet. We recall that $f \leq \Omega(\mathbf{k}) \leq N$, where f and N are the Coriolis and local buoyancy frequency. A initial time $t = 0$, we assume that the wave induced energy is confined within a two-dimensional Gaussian envelope along the y and z directions.

RAY EQUATIONS AND DIRECT NUMERICAL SIMULATIONS

We solve the classical ray equations (see *f.i.* [1]), which describe how a wave vector is refracted by the gradients of the background flow along a ray. From a practical point of view, the ray equations are initialized by a set of rays starting from points that model the wave packet. The wave vector and intrinsic frequency at each of these starting points are computed using the constancy of k_x and of the absolute frequency along a ray. We also solve the Navier-Stokes equations in the Boussinesq approximation in a parallelepipedic domain. The boundary conditions are periodic along the x and z directions and of free slip type along the y direction, so that a pseudo-spectral method can be used. The equations are integrated in time using a third-order Adams-Bashforth scheme.
We have performed several computations, which are described in [2]. The point of view we have chosen is the following: the wave packet propagates into the current such that its intrinsic frequency increases because of the y-dependency of the shear flow. This means that the wave packet should be trapped in the neighbourhood of the $\Omega = N$ surface and possibly break there, at least when $\beta = 0$ [3], thereby inducing mean flow changes. Our purpose is to investigate the influence of the baroclinicity parameter β on this behavior for $0 < \beta \leq 1$.

RESULTS

We illustrate the wave-shear flow interaction for a background flow with a weak horizontal shear, with Rossby number (close to 0.3) comparable to that of the wave packet at initial time ($simeq 0.85$). The ratio $N_0/f = 4.4$ (where $N_0 = 1$ is the buoyancy frequency of the fluid at rest) and the Prandtl number is equal to 1. Two cases are presented, corresponding to $\beta = 0.5$ and $\beta = 1$.

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Ray trajectories predicted from ray theory for $\beta = 0.5$ are displayed in Figure 1a. The rays steepen in the neighbourhood of the $\Omega = N$ surface, are trapped there and travel downwards with the now quasi-vertical group velocity. The intrinsic frequency Ω increases during this stage (Figure 1b). The shear flow we have designed possesses regions where $\partial U/\partial z$ vanishes, U and $\partial U/\partial y$ being minimum there, so that the $\Omega = N$ surface flattens about this region. This is where the rays, which have become quasi-vertical because Ω is close to N, reflect. Figure 1a shows that the rays are able to propagate further in the shear flow, within a wave guide limited by two portions of the $\Omega = N$ surface. Results from DNS for the same run are plotted in Figures 1c to 1e. The wave amplitude first decays because of dispersion. This is an important effect, which the subsequent interaction with the shear flow (through which the wave amplitude increases) does not make up for. As in the ray theory, the wave packet is then steepened by the trapping process (Figure 1d), travels along the $\Omega = N$ surface and meets again that surface when it flattens (Figure 1e). But as opposed to the ray theory predictions, no reflection is clearly observed: the decay of the wavelength along the y-direction and of the group velocity makes the packet very sensitive to viscous effects. It has totally dissipated by the time the rays first reflect ($t \simeq 300$), assuming the ray theory predictions are quantitatively reliable.
When the baroclinicity parameter $\beta = 1$, U and $\partial U/\partial y$ now also vanish in the region where $\partial U/\partial z = 0$. Figure 1f shows that the wave packet now penetrates into the shear flow, because the trapping process has been suppressed, and that this region acts as a wave guide.

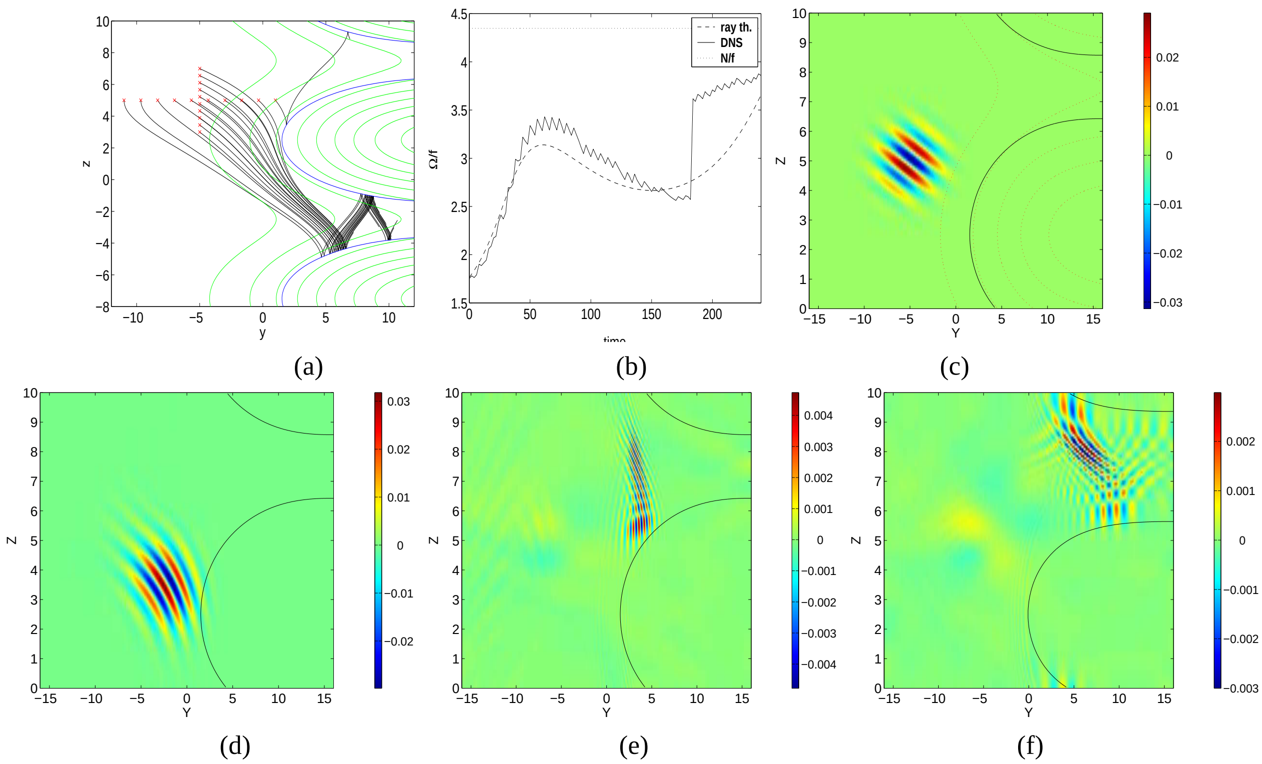

Figure 1. (a) to (e): $\beta = 0.5$. (a) Predictions from ray theory; trajectories of rays starting from points modelling the initial wave packet at $t = 800$. (b) Intrinsic frequency computed at the packet center versus time, from ray theory and DNS. (c) to (f): Constant contours in a vertical (y, z) plane of the fluctuating density field computed from DNS, at $t = 0\ (c),\ 56\ (d),\ 84\ (e),\ 176\ (f)$. (g) Run $\beta = 1$. DNS results at $t = 204$. In all frames (but b)), the surface $\Omega = N$ is displayed with a blue line.

CONCLUSION

When a wave packet propagates in a baroclinic shear flow such that its intrinsic frequency increases due to the horizontal shear, DNS of the Boussinesq equations show that the packet amplitude is first attenuated because of dispersion, then is trapped in the vicinity of the $\Omega = N$ surface and is dissipated there. When the shear flow locally vanishes, both ray theory and DNS results predict that the wave can penetrate into the shear flow through a wave guide. The resulting changes of the background medium are however insignificant if the wave is not forced. Our study suggests that there should be "less" waves inside large scale geophysical vortices than outside.

References

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