SPIRAL VORTEX BREAKDOWN AS A GLOBAL MODE

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Summary The spiral form of vortex breakdown observed in the numerical simulations of Ruith, Chen, Meiburg and Maxworthy[5] is interpreted as the consequence of the development of a so-called nonlinear global mode originating in the convective/absolute transition of the instability in the lee of the vortex breakdown bubble. This local theory gives an excellent prediction of the precession frequency measured in the three-dimensional DNS.

MOTIVATION

Vortex breakdown is a widespread phenomenon affecting swirling jets that is both observed in its axisymmetric form (the bubble) or in its spiral form, as soon as the swirl parameter S, which compares the intensity of the azimuthal velocity component to its axial counterpart, is large enough. These synchronized helical vortex breakdown states (see Escudier [1]) for a review) are characterized by low azimuthal wavenumbers ($m = 1$ or $m = 2$), they rotate in time in the same direction as the swirling base flow but wind in space in the opposite direction.

The aim of the present study is to show that these non-axisymmetric vortex breakdown states observed at moderate Reynolds numbers may be interpreted as the result of local instability of the unstable axisymmetric breakdown serving as a spatially varying base flow. More precisely, according to the initial suggestion of Escudier, Bornstein and Maxworthy[2] and Delbende, Chomaz and Huerre[3], the spiral vortex breakdown is supposed to result from the development of a global mode triggered by the absolutely unstable nature of the $m = 1$ mode in the lee of the breakdown bubble. Yin, Sun, Wei and Wu[4] and Ruith *et al.*[5] have indeed shown that the spatio-temporal stability results associated with the $m = 1$ mode of the Batchelor vortex compare qualitatively well with the experimental characteristics of spiral vortex breakdown states.

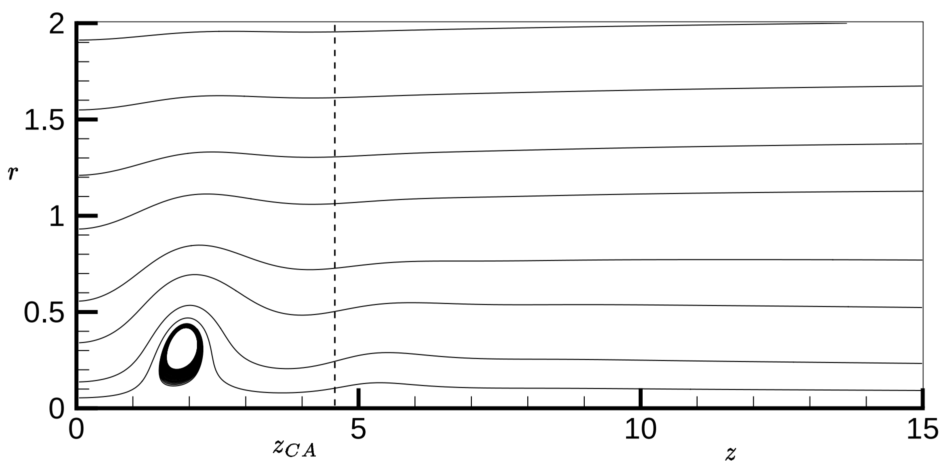

Figure 1. Meridional cut of the streamlines associated to the base flow obtained numerically by imposing the axisymmetry of the flow; $S = 1.095$, $Re = 200$ (Ruith *et al.*[5]).

Our purpose is to validate this hypothesis by using as a base flow the velocity profiles obtained in the numerical simulations of Ruith *et al.*[5] by imposing axisymmetry (see figure 1). As seen in figure 1, vortex breakdown is observed at the swirl number $S = 1.095$, characterized by the typical recirculation zone. At each axial station z along the streamwise axis, the flow is assumed to be weakly non-parallel and the stability study is conducted on a parallel flow basis with the base flow measured locally at the station z. The weakly non-parallel but strongly nonlinear theory of Pier, Huerre and Chomaz[6] suggests that, if there exists a station $z_{C/A}$ where the imaginary part of the absolute frequency ω_0 vanishes and where the flow changes from convective ($\omega_{0,i}(z) < 0$ for $z < z_{C/A}$) into absolute ($\omega_{0,i}(z) > 0$ for $z_{C/A} < z$), then a nonlinear global mode might be triggered with a front located at $z_{C/A}$. This global mode also inherits the real absolute frequency at this point $\omega_0(z_{C/A})$.

COMPARISON BETWEEN WEAKLY NON-PARALLEL STRONGLY NONLINEAR THEORY AND THREE-DIMENSIONAL DNS

For the same parameter setting as in figure 1 but by relaxing the axisymmetry assumption, figure 2 represents the instantaneous streaklines at $t = 1850$ obtained in the DNS with an initial random noise imposed on the base flow of figure 1 at

$t = 0$. The flow settles to a limit cycle oscillating at a well determined frequency ω_G^{NL}.

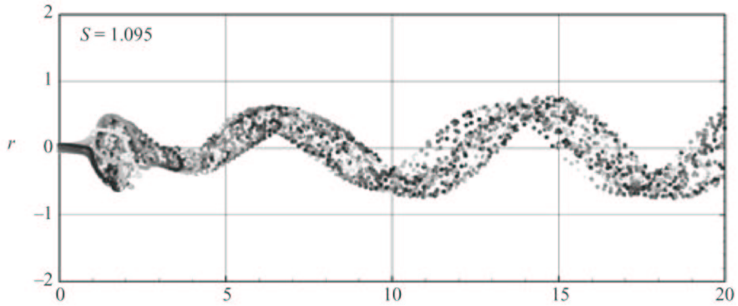

Figure 2. Instantaneous streaklines at $t = 1850$ associated to the three-dimensional flow obtained by DNS at $S = 1.095$ and $Re = 200$ (Ruith *et al.*[5]).

The stability analysis is conducted by numerical means using the method outlined in Delbende *et al.*[3] in a box $1440 \times 196 \times 196$. The results are summarized in figure 3. Two absolutely unstable regions can be distinguished as indicated by the shaded areas. A first region located in the recirculation bubble between $z^1_{C/A} = 1.1$ and $z^1_{A/C} = 3.3$ and a second region in the lee of the bubble for $z \geq z^2_{C/A} = 4.7$.

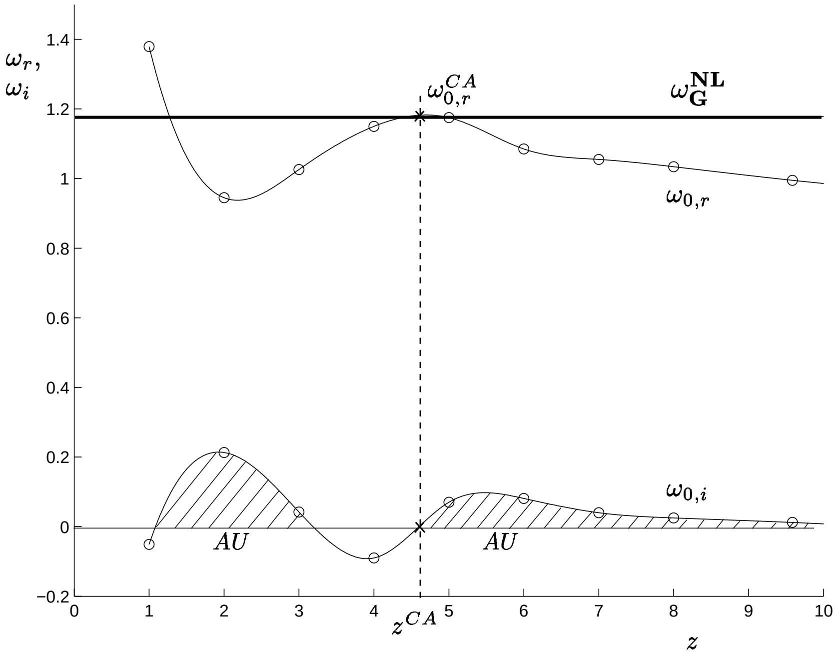

Figure 3. Streamwise evolution of the real $\omega_{0,r}$ and imaginary part $\omega_{0,i}$ of the local absolute frequency as a function of streamwise distance z.

The value of the global frequency ω_G^{NL} measured in the DNS is shown in figure 3 as a horizontal thick line. The agreement between the measured frequency ω_G^{NL} and $\omega_{0,r,2}^{C/A}$ is excellent, whereas it is less satisfactory with $\omega_{0,r,1}^{C/A}$. This shows that the second region is responsible for the global instability and that the transition point $z^2_{C/A}$ plays the role of a wavemaker. The validity of the interpretation of spiral vortex breakdown in terms of a nonlinear global mode with a front located at the transition point $z^2_{C/A}$ is further confirmed by the analysis of the streamwise amplitude distribution and wavenumber variation extracted from the DNS.

References

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