TRANSITION TO CHAOTIC MARANGONI CONVECTION IN LIQUID BRIDGE

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Summary Marangoni convection is investigated in cylindrical column using a liquid with Pr=4. The present results are targeting on the study of the non-linear characteristics of the flow under zero-gravity conditions. The transitions to periodic, quasi-periodic and chaotic flows are investigated numerically. The 3-D oscillatory flow is a result of a supercritical Hopf bifurcation and the periodic orbit represents the unique stable solution near the onset of the instability. The non-linear system admits regime of bi-stability. A traveling wave with azimuthal wave number m=2 bifurcates from the basic branch of axisymmetric steady state; this branch remains stable in the considered range of parameters. A second stable branch with azimuthal wave number m=3 appears for higher Marangoni numbers and reveals other periodic, quasi-periodic and chaotic properties. The transitions between the two stable orbits with m = 2 and m = 3 have never been observed.

INTRODUCTION

An increasing number of experimental studies in a half-zone model, which corresponds to floating zone (FZ) techniques of crystal growth, indicates that convection in melt should be turbulent, e.g. see Hurle[1]. Nevertheless, because of great complexity of the turbulent flows, all numerical simulations of transport processes were performed assuming laminar flow in the liquid phase. Apart of that, from the more physical side, the hydrodynamics effects in the half-zone model is of basic interest for the dynamics occurring in the system, as it is an excellent example of a dissipative dynamical system. Therefore, the present study is aimed at the investigation of time-dependent convective flows in the strongly supercritical regimes.

FORMULATION OF THE PROBLEM

A liquid bridge consists of a fluid volume, which is held between two differentially heated horizontal flat concentric disks of radii R, separated by a distance d. The temperatures T_h and T_c ($T_h > T_c$) are prescribed at the upper and lower solid-liquid interfaces respectively, yielding a temperature difference $\Delta T = T_h - T_c$. The surface tension and kinematic viscosity are taken as linear functions of temperature. Throughout this parametric study the Prandtl number and the aspect ratio $\Gamma = d/R$ are kept constant; Pr=4, Gr=0 and $\Gamma = 1$. Then the only one parameter, the Marangoni number, which is proportional to the temperature difference between the rods, controls the flow.
The governing Navier-Stokes, energy and continuity equations are written in non-dimensional primitive-variable formulation in cylindrical co-ordinate system, see details in Shevtsova et al[2]. The three-dimensional, fully non-linear governing equations were solved in a primitive-variable form on a staggered stretched mesh. These equations were integrated over non-overlapping finite volumes. The computation of the velocity field at each time step was carried out with the projection method. A combination of fast Fourier transform in the azimuthal direction and of an implicit ADI method in the others was applied for calculating the Poisson equation for pressure.

RESULTS AND DISCUSSION

The different spatiotemporal patterns of the thermocapillary flow are numerically analyzed, beginning from the onset of instability up to appearance of non-periodic flow and further on. The calculations were performed up to $\varepsilon = (\Delta T - \Delta T_{cr})/\Delta T_{cr} \approx 8.5$. For the small temperature difference between supporting disks the thermocapillary flow is two-dimensional (i.e. invariant in the azimuthal direction), steady and has a toroidal-like structure. The 3-D oscillatory flow is a result of a supercritical Hopf bifurcation and the periodic orbit (traveling wave with azimuthal wave number m = 2) represents the unique stable solution near the onset of the instability, $Ma_{cr} = 2520$. This periodic branch remains stable in the considered range of parameters. A second branch of solution with azimuthal wave number m=3 appears for higher Marangoni numbers, $Ma_{cr}^{(2)} = 3240$, and reveals other periodic, quasi-periodic and chaotic properties.
Beyond this second bifurcation point, Ma >3240, the system admits the coexistence of two stable oscillatory solutions with two different wave numbers, m = 2 and m = 3. The transitions between the two stable orbits with m = 2 and m = 3 have never been observed. Notice, that these two solutions do not represent the different modes of the linear problem; on the contrary, they are both the results of the solution of the full non-linear problem. The final solution depends on the initially chosen wave number guess.

The symmetry of the final solution keeps the memory of the initial state of the system for Ma> $Ma_{cr}^{(2)} = 3240$. Namely, taking an initial guess with m = 1, 2, 4, 6 etc. symmetries, after some transient time the system will arrive to m = 2 traveling wave (TW) solution. Otherwise, all the odd basic symmetries m = 3, 5, 7 (except m = 1) give m = 3 traveling wave as final state of the system.

For the m = 2 solution, the critical mode at the first bifurcation, the flow remains strictly time-periodic for $\varepsilon < 8.5$.

The m=3 flow pattern undergoes a transition from the periodic to a weakly chaotic flow via quasi-periodic and period-doubling states. This study is extension of recently published results, see ref.3, where all above-mentioned temporal transitions have been analysed. Here the special attention is paid to the investigation of the *spatial* structure of the flow with increasing of Marangoni number. The results of spatial and temporal Fourier analysis are shown in Fig.1 and Fig.2 for Ma= 18000, while temporal chaos is progressing.

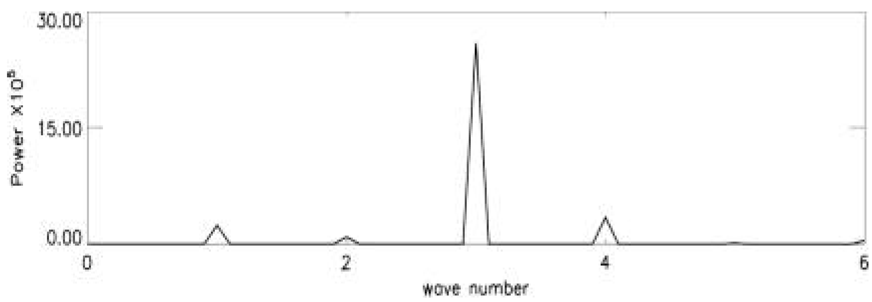

Fig.1. Spatial Fourier spectrum, Pr=4, Ma=18000.

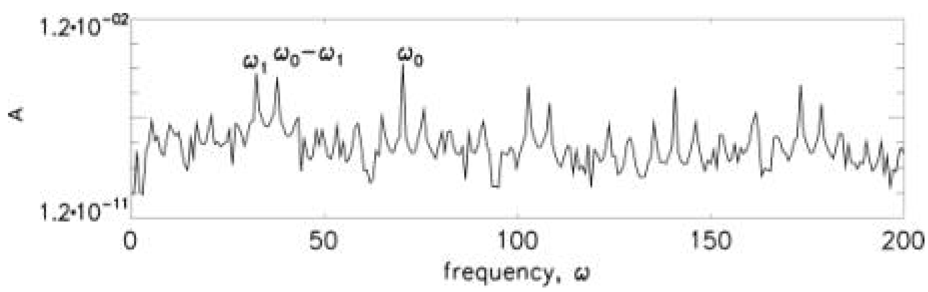

Fig.2. Temporal Fourier spectrum, Pr=4, Ma=18000.

To distinguish numerically the chaotic and non-chaotic (aperiodic) behaviour is a delicate problem. There are several ways to recognize chaotic behaviour features in oscillatory systems, such as analysing their Poincare (return) maps, phase space trajectories and power spectra. One of the fundamental characteristics of a chaotic state is its sensitivity to the initial state. If the same system starts from two different but close initial conditions (initial guesses in our numerics) their dynamical trajectories would diverge on the attractor very quickly. The distance between two close trajectories has to grow exponentially with time.

Of course, the simulation of the linear equations obtained by linearization of the Navier-Stokes equations around the non-linear solution would give the exponential law more directly. Nevertheless, very small disturbances should evolve in the exponential way even if one simulates the original Navier-Stokes equations. For the present system, the difference between original and different disturbed solutions were carefully analyzed to demonstrate that temporal aperiodic flow is chaotic.

CONCLUSIONS

The development of Marangoni convection in liquid bridge has been studied in 3D numerical model. To the best of our knowledge, the bifurcation of thermocapillary flow in a liquid bridge in the strongly supercritical regime has not yet been mapped out. Tracing the route to temporal chaos, two attractors corresponding to traveling waves with different wave numbers have been found. One of the attractors exhibits temporal chaotic behaviour, while the other one is strictly periodic. The spatial structure of the flow does not reveal chaotic features.

References

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