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INCREMENTAL NONLINEARITY IN CONSTITUTIVE RELATION FOR GRANULAR MEDIA

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Summary In this paper, the 3D Granular Element Method was utilized to conduct stress probe tests. It was found from these tests that, as far as the conventional tri-axial state concerns, the incremental plastic response is approximated by the non-associated flow rule, and that, for general true tri-axial stress probes, the direction of incremental plastic strain is apparently dependent on the direction of incremental stress. The latter suggests the existence of multiple shear mechanisms in plastic deformation of granular media.

INTRODUCTION

Recently, incrementally nonlinear models have been proposed for constitutive relations of geomaterials[1], [2]. In these theories the direction of incremental plastic strain is generally dependent on the direction of incremental stress. The dependency of the direction of incremental plastic strain on the direction of incremental stress was experimentally demonstrated in a series of torsional shear tests on hollow cylindrical specimens of sand [3]. As the principal directions of stress in these tests were not fixed, it was concluded that the incremental nonlinearity emerged from the rotation of principal axes of stress. For general discussion, we need to develop a sophisticated testing apparatus which enables us to carry out arbitrary probe tests in six dimensional stress or strain space and conduct a huge series of tests of numerous specimens regarded as identical. On the other hand, numerical tests with a suitable discrete element method enable us to extract general information on irrecoverable behaviors of an identical granular specimen. The paper [4] tried to clarify the 3D incremental responses numerically, using the Granular Element Method (GEM) proposed by the author. However, it dealt with only tri-axial stress probes and the incremental nonlinearity was not remarkable. This paper, based on the GEM numerical tests under the true tri-axial state, discusses the incremental nonlinearity in the plastic flow of granular media.

METHODS

The discussions in this paper are based upon numerical element tests of a spherically packed assembly of spherical particles. The numerical experiments are a triaxial compression test and two series of stress probe tests. A stress probe test is a set of incremental loading and reversal loading along a specified stress probe direction, which gives the incremental elastic strain as the recoverable part and the incremental plastic strain as the irrecoverable part. All the loadings are controlled by the stress which can be determined from contact forces between peripheral and inner particles. The Granular Element Method(GEM), which is basically a stiffness method, makes it easy for boundaries to be controlled in arbitrary ways. The movement of each inner particle (parallel displacement and rigid rotation) has six degrees of freedom, while the non-rotational movement of a boundary particle follows a linear transformation determined by the bulk deformation. The relationship between contact force and relative displacement at each contact point is characterized as in DEM by spring coefficients in normal and tangential directions and Coulomb's friction law and the no-tension rule are assumed. The algorithm in GEM utilizes an implicit iterative procedure to secure accurate equilibrium states. Each step-by-step particle movement for unbalanced force and moment is determined by an individual stiffness matrix with 6 x 6 components assembled under the assumption that the neighboring particles do not move. The latter idea has stemmed from a relaxation analysis for elastic frames. However, in non-elastic analyses, the history of relaxation in an iterative process affects the results. Hence, in GEM, all of the particle movements in an iteration step take place at the same time after determining individual particle movements by stiffness matrices. If a particle does not have three contact points, the stiffness matrix of this particle becomes singular. In the GEM algorithm, a singular stiffness matrix is replaced with a diagonal matrix whose components have small values. To avoid excess overlapping between particles, the particle movements calculated by the individual stiffness matrices are modified such that the maximum movement does not exceed a limit.

RESULTS

The numerical specimen used in this research is a random assemblage of 360 spherical particles whose radii range from 1.3 mm to 2.6 mm and 166 particles among them are peripheral particles. Spring constants for normal and tangential directions are 2.0 and 1.4 MN/m, respectively and the friction angle is 15^o. As shown in **Figure 1.**, the tri-axial test starts from a state with isotropic pressure of 0.1 MPa and this value is maintained in the horizontal directions throughout the tri-axial compression test. The tolerance in the boundary stress control is 1.7×10^{-7} MPa. Such small value is required to determine both of the elastic and plastic strains with sufficient accuracy. Stress probe tests are carried out starting from an intermediate state of tri-axial test whose stress ratio is 0.4. All of the stress probe directions lie in the principal stress space, and a series of stress probe tests are a set of 72 probes in a plane of stress space. The magnitude of each stress probe was 0.001 MPa which is one hundredth of the initial confining pressure. To specify probe directions, we define three characteristic directions based on a yield surface (**Figure 1.**). We assume that the yield surface is a cone whose vertex is at the origin of the principal stress space. The validity of this assumption will be demonstrated later by the results for tri-axial loading. We will also find later that we cannot assume a unique yield surface for general incremental

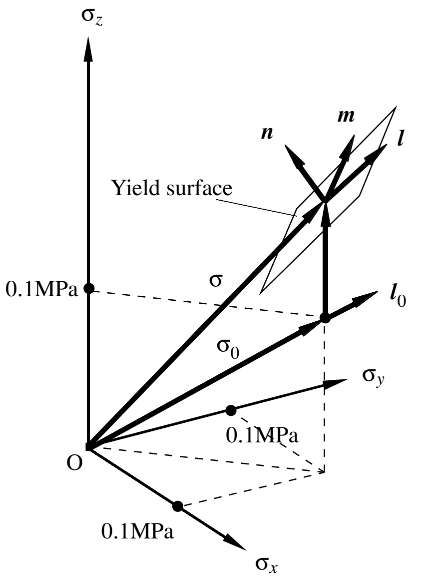
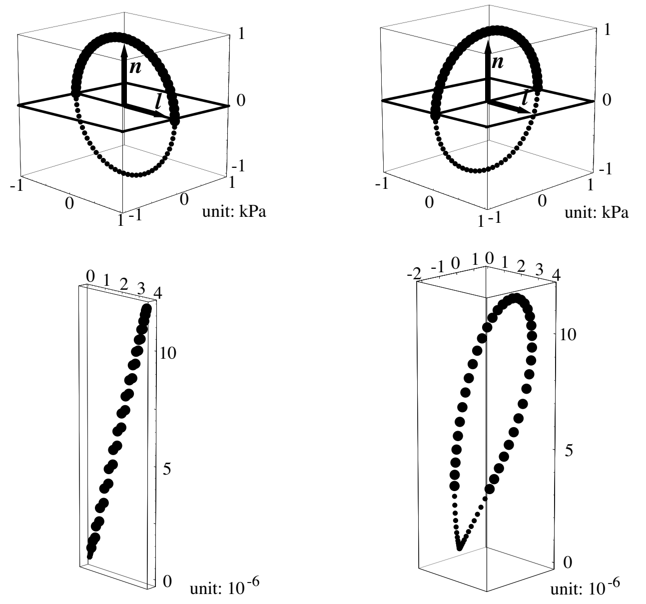


FIgure 1. Loading path



Probe test (1) Probe test (2)

Figure 2. Plastic strain increment

stresses. However, we define characteristic directions by assuming that the yield surface is the cone explained above. The first characteristic direction ***n*** is the outward normal of the yield surface. The second direction ***l*** is the direction of current stress vector which is a generator of the current yield surface. The last direction ***m*** is perpendicular to these two directions. Two types of stress probe tests are carried out. As shown in **Figure 2.**, the probe planes for these probe tests are (***n***, ***l***) and (***n***, ***m***) planes. The stress state in Probe test 1 always remains in the so-called tri-axial state, while, in Probe Test 2, the stress state is generally in the so-called true tri-axial state. **Figure 2.** shows the results obtained in the stress probe tests. Stress probes and incremental strains are plotted in the stress and strain spaces. The incremental response for a stress probe that has downward normal component with respect to the yield surface is usually assumed to be elastic, and in fact the incremental plastic strains in Probe test 1 are almost zero as shown by the smaller dots. The result is also in accord with the previous assumption that the yield surface is given by a cone whose vertex is at the origin of the principal stress space. Further, in Probe test 1, it is observed that the incremental plastic strain vectors are almost on a unique straight line. As this line is not coaxial with the outward normal of yield surface, the plastic flow is non-associative. On the other hand, the incremental plastic response in Probe test 2 is totally different from the above result. The incremental plastic strain vectors are not on a common line and the direction of incremental plastic strain is apparently dependent on the direction of incremental stress. Further, the smaller dots represent incremental plastic strains for stress probes whose directions are inward of the assumed yield surface, which indicate that we can not assume the existence of a unique yield surface generally. From these results, we may conclude that the plastic deformation is generally accompanied by multiple shear mechanisms. One of the definite mechanisms is that of tri-axial shearing which has been developing during the tri-axial compression. Another mechanism begins to develop when the shearing between two horizontal stress components is applied to the specimen for the first time in Probe test 2.

CONCLUSIONS

In this paper, the numerical tests were carried out to verify the incremental nonlinearity in constitutive relationship for granular media. The method used was the 3D Granular Element Method, and the simulation of true tri-axial probe tests led to the following results: 1) As far as the conventional tri-axial state concerns, the incremental plastic response is approximated by the non-associated flow rule. 2) For general true tri-axial stress probes, the direction of incremental plastic strain is apparently dependent on the direction of incremental stress. 3) The latter suggests the existence of multiple shear mechanisms in plastic deformation of granular media.

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