XXI ICTAM, 15-21 August 2004, Warsaw, Poland

EXPRESSION ON THE DEFLECTION OF A FLEXIBLE THIN ROD AND IT'S MEASUREMENT

Kohichi Miura*, Tetsu Nishimura *
* *Nihon University, Department of Mechanical Engineering, College of Science & Technology, Tokyo, Japan*

Summary A purpose of this investigation is to discuss the suitable expression of motion of flexible body for experimental observation. The authors have intended to establish an expression of motion of flexible body system which consists of interconnected rigid and deformable components. In this paper, a deformable body element which is very long in comparison with its width and have a uniform circular cross section, such as a flexible rod, flexible beam, or cable element, are investigated specifically.

INTRODUCTION

Many mechanical systems which consist of interconnected components, so called a multibody system, are able to perform variable and large transformational and rotational displacements. The motion of component also is kinematically restricted because of different types of connecting joints. Hence each subsystem included in the system behaves according to the specific nature of the transformational and rotational displacements. The procedures on a dynamic analysis of multibody system are mainly developed for many mechanical systems such as robots, vehicles, aircraft, space structures and machine mechanisms. The expression of motion on kinematics is established in this field with the intention to control the mechanical systems. However these approaches are not suitable for grasping a broad view of motion for a flexible body, because the rotation of the whole system is difficult to define especially.

It is necessary to discuss the suitable expression of motion of flexible body for experimental observation. As an example, a deformable body element, which is very long in comparison with its width and have a uniform circular cross section, such as a flexible rod, flexible beam, or cable element, are investigated. Many mechanical systems such as industrial robots, automated production systems, and crane systems which containing cable elements, power cable, a distributing cable and cable carrier are able to perform variable and large transformation and rotational displacements.

Along with a high speed or performances on these mechanical systems, these cable elements occasionally meet with the severe condition. In these cases, cable elements are strained by bending deformation accompanied with a torsional deformation. Here, the bending and torsional deformation within the element is discussed. It is assumed that the cross section normal to its longitudinal axis remains plane after deformation, and axial tension is disregarded. In this case, the problem what to measure the position of centroid of body element arises as mentioned before. Since it is difficult to estimate directly the torsion and the curvature of deformable cable elements, to obtain a broad view of the motion is an effective measurement.

MEASUREMENT METHOD

In this study, useful methods are investigated for the kinematical measurement of torsion and curvature of a circular cable element, which is observed by two high-speed video cameras. (See Fig.1) In order to investigate a motion analysis of a deformable circular cable element, a family of compact formulation of bending and twisting for a deformable circular cable element is introduced. The measurement of cable element, suggested here, may be available for a motion analysis of cable structures. If the mass distribution is variable, measurement of the centroid is exceedingly complicated furthermore. However, the centroid of deformable body moves whenever the element deformation occurs, it is very difficult to measure the centroid of deformable body element.

To measure the deformation of thin rod element, the circumferential marking lines are graduated in equal distance along longitudinal direction. In order to express the bending with sole curvature, the longitudinal distance is properly selected as L, which depends on its diameter and stiffness. Moreover, the longitudinal marking lines are positioned equally around the circumference. Then, intersections of circumference and longitudinal lines are the target points to measure. If the positions of three points on the circumference of the rod element are measured, the position of center of cross section and normal direction on the measuring cross section are obtained from a geometrical consideration. For the purpose of obtaining the three-dimensional position data, it is necessary to observe the rod element from two directions. Considering a blind spot, eight or more points are required along the circumference line in order to obtain the three position data on the measuring cross section.

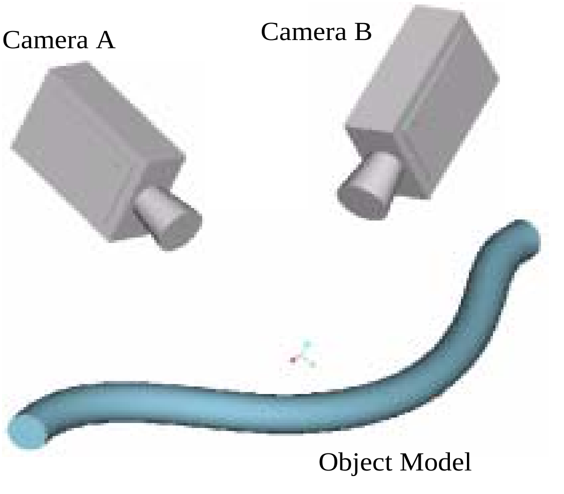

Fig.1 MEASURING IMAGE

One end plane of component, that is the unit element, is the base plane, and another plane is rotational plane (See Fig.2). Searching the relation of transformation between the body-coordinates on the base plane and the rotational plane, the deformation is resolved. By measuring the position and the direction on both end planes of the component, the curvature of deformed rod component is determined geometrically. And, the profile of the deformed rod component is approximated by an arc with the obtained curvature. As a necessary consequence, the position of the centroid and the inertia tensor of the rod component are able to determine by the arc profile. Finally, the position of the centroid and the inertia tensor of whole rod element are obtained. From the measurement of deformation and the measurement of motion, the angular motion of flexible body system containing deformable elements becomes clear. The authors have intended to establish an expression of motion of flexible body system which consists of interconnected rigid and deformable components. A family of compact generalized formulations of rotational motion for a flexible body or multibody has been derived by adopting an instantaneous rotational axis of the whole body.[1]

For general deformable elements which have a few constraints on the deformation, it is difficult to determine the deformed profile; its analysis requires a special technique.

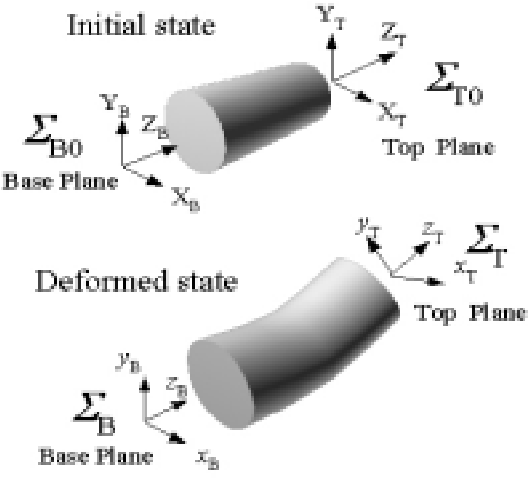

Fig.2 MEASURING MODEL

EXPRESSION ON THE DEFLECTION

In the actual methods for the dynamic analysis of multibody system, the expression of angle variables are required in order to define the orientation of the multiple body. Some of the most commonly used parameters for expressing the angle variables are Euler angles, Euler parameters, navigation angles and the direction cosines. These variables are used to consider the physical meaning of angular parameter set which correspond to the degree of freedom at each joint or motion of the system. However these expressions are not suitable for grasping a broad view of motion and deformation for cable, because it is difficult to estimate directly the torsion and the curvature of deformable cable elements.

To consider a deflection of the element, the local coordinate system Σ_T located on the top plane is observed in the fixed coordinate systemΣ_{B0} located on the base plane (See Fig.2). In initial state, the local coordinate system Σ_T coincide with the fixed coordinate systemΣ_{B0}. After deformation, the coordinate system located on the top plane is denoted Σ_{BT}. At first, the coordinate system Σ_{BT} is swiveled through ω about Z_B axis. Next, Σ_{BT} is swiveled through θ about the axis which is turned Y_B axis round the Z_B axis by an angle ϕ. Obtained coordinate system is rewritten by an expression in the fixed coordinate system. Using the coordinate transform matrix, Σ_{BT} is expressed by the following equation.

$$\Sigma_{BT} = \mathbf{R}_{\Phi}\mathbf{R}_{\Theta}\mathbf{R}_{\Phi}{}^{T}\mathbf{R}_{\Omega} \tag{1}$$

A noteworthy finding is that ω is the angle of twist of element and the physical meaning of θ is considered to be the normal curvature. Furthermore, ϕ define the binormal vector of element.[2] Using the set of angular parameter introduced here, the transformation matrix can be multiplicatively decomposed into the product of a matrix due to bending deformation and a matrix due to twisting deformation. This expression is possible to draw a clear distinction between bending and twisting for experimental operation.

CONCLUSIONS

The expression on the deflection of a cable element and its measurement, suggested here, may be available for a dynamic analysis of cable structures or many mechanical systems which containing cable elements. These approaches are effective in supplementary experiment to the main experiment with electric measurement.

References

[1] K.Miura, T.Yamada, K.Kawamura & T.Nishimura: Expression of Angular Rotation of a Multibody System -Application to a rotational expression of a flexible body-. 6th International Conference on Motion and Vibration Control, Vol.1, pp520- 525

[2] A.E.H.LOVE.: A Treatise on The Mathematical Theory of Elasticity (fourth edition). Dover Publications, 1927.