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A THEORETICAL MODEL FOR RESONANCES IN FLOW PAST A CAVITY

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Summary Cavity resonances involve a coupling between instability waves on the shear layer across the face of the cavity and acoustic waves that propagate between the cavity ends. The instability and acoustic waves are coupled by the scattering processes at the cavity ends. We utilize the Wiener–Hopf technique to analyze these scattering processes. Combining the scattering solutions with propagation solutions for the central region of the cavity, we obtain a global model whose predictions are in good agreement with experimental data.

INTRODUCTION

Acoustic resonances leading to high unsteady pressure levels may occur in flow past cavities. These are of concern in a variety of applications, including military aircraft weapons bays. The resonance involves a coupling between the downstream-propagating instability wave on the shear layer spanning the open face of the cavity, and acoustic waves propagating within and external to the cavity. These elements of the disturbance field are coupled by scattering processes at the upstream and downstream ends of the cavity.

A seminal contribution to the understanding and prediction of cavity resonances was made by Rossiter [1], who performed an extensive experimental study of cavity resonances for free-stream Mach numbers in the range $0.5 \leq M \leq 1.2$, and also proposed a semi-empirical formula for the resonant frequencies that remains widely used even today. Rossiter's formula is based upon an $n2\pi$ phase shift around a simple resonant loop consisting of a downstream-propagating instability wave and an upstream-propagating acoustic wave. Rossiter introduced an empirical constant to account for the phase shifts associated with the scattering processes at the two ends of the cavity, which couple the instability and acoustic waves. Heller & Bliss [2] presented experimental results over an extended Mach number range $0.3 \leq M \leq 3.0$, and proposed a modified Rossiter formula in which the static temperature in the cavity is assumed to be the stagnation temperature of the stream. The importance of this effect increases with Mach number.

Tam & Block [3] carried out an extensive experimental investigation at low Mach numbers. They also developed a theoretical model which involved excitation of the shear layer instability wave by the acoustic field generated at the downstream end of the cavity. The scattering process at the downstream end was represented in a simplified fashion, by introducing a concentrated acoustic monopole source at the downstream corner of the cavity. The phase of the source relative to the local instability wave motion was chosen heuristically, in a manner similar to the empirical constant of Rossiter's formula. The excitation of the shear layer instability wave was then calculated through a 'distributed receptivity' model, which ignored details of the interaction at the upstream end of the cavity. The amplitude of the monopole source was not determined. Therefore, the theory produced an expression for the resonance frequencies (with some empirical input), but did not determine whether a particular resonant mode corresponded to a globally stable or unstable disturbance.

Cavity resonances have received increased attention recently. This recent research has involved primarily numerical simulations and experiments, with a significant focus on active control. Many recent contributions can be found in the acoustics and fluid mechanics journals.

Thus, previous theoretical models for cavity resonances have generally provided only incomplete representations of the phenomenon. In particular, the scattering processes at the cavity ends have generally not been analyzed, or have been treated heuristically. The phases of the scattering coefficients at the two ends of the cavity are required for a first-principles prediction of the resonant frequency, while the amplitudes of the scattering coefficients are required in order to determine the temporal growth (or decay) rate of the mode, and the instability boundaries of the mode in parameter space.

ANALYSIS

In this paper, we present a model for cavity resonances that incorporates theoretical predictions for the scattering coefficients at the two ends of the cavity. The scattering coefficients are determined by local solutions which treat the square-corner geometry exactly. The scattering coefficients are then combined with theoretical predictions for the propagation of the shear-layer instability wave and the acoustic waves along the length of the cavity. The combined system takes a form similar to a matrix eigenvalue problem, where the eigenvalue is the complex frequency. The real part of the frequency determines the temporal oscillation of the unsteady field, while the imaginary part of the frequency determines the temporal growth (or decay) rate of the mode.

Consider a two-dimensional rectangular cavity of length L and depth d under a stream of speed U, as illustrated in figure 1. The external stream is separated from the quiescent fluid in the cavity by a shear layer. The total field is the sum of a mean field and a time-harmonic perturbation field with complex frequency ω. For the case of a supersonic stream, the time-harmonic perturbation field consists of a shear-layer instability wave of (complex) amplitude $\mathcal{S}$, upstream- and downstream-propagating cavity acoustic modes of amplitudes $\mathcal{U}$ and $\mathcal{D}$, and 'fast' and 'slow' wave fields in the supersonic stream of amplitudes $\mathcal{E}_f$ and $\mathcal{E}_s$. Note that $\mathcal{S}$, $\mathcal{D}$, $\mathcal{E}_f$ and $\mathcal{E}_s$ are all downstream-propagating components. For a supersonic stream, the only upstream-propagating component is $\mathcal{U}$. The elements of the unsteady field for a supersonic stream are illustrated in figure 1. For the case of a subsonic stream, the external field components are replaced by a (sin-

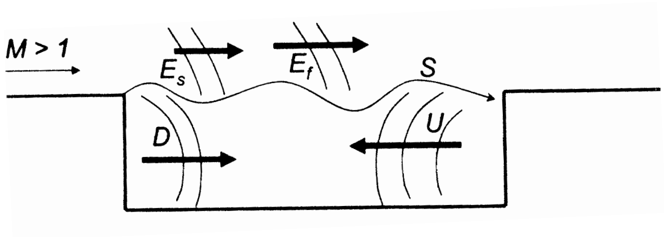

Figure 1. Illustration of the global model for a cavity acoustic resonance in supersonic flow.

gle) downstream-propagating wave field $\mathcal{E}_d$ and an upstream-propagating wave field $\mathcal{E}_u$. We have developed theoretical prediction methods for both supersonic and subsonic streams. However, for the sake of brevity, in describing the analysis we shall focus on the case of a supersonic stream.

For the scattering process at the upstream end, the incident field is the upstream-propagating cavity mode $\mathcal{U}$, while the scattered field contains $\mathcal{S}$, $\mathcal{D}$, $\mathcal{E}_s$ and $\mathcal{E}_d$. At the downstream end, there are four separate incident fields ($\mathcal{S}$, $\mathcal{D}$, $\mathcal{E}_s$ and $\mathcal{E}_d$) and a single scattered field ($\mathcal{U}$).

The scattering process at each end is analyzed separately. The analysis involves the following steps. First, the end wall of the cavity is removed and the case of a semi-infinite overhanging lip is considered. The geometry is then amenable to application of the Wiener–Hopf technique [4]. The scattered field produced by impingement of the incident field consists of an infinite set of acoustic cavity modes that propagate back into the cavity, an infinite set of acoustic duct modes that propagate into the 'duct' formed by the cavity bottom and the semi-infinite overhanging lip, and an external acoustic field in the stream. For the scattering process at the upstream end, a shear-layer instability wave is also generated.

A second scattering problem for the case of a semi-infinite overhanging lip is then solved, in which the incident field is an acoustic duct mode propagating toward the cavity. This also generates a scattered field containing the same components as described in the preceding paragraph.

The solution for a finite-length overhanging lip geometry, including the presence of the end wall, is then obtained by noting that the complete field under the overhanging lip is composed of an infinite set of upstream- and downstream-propagating duct modes. Applying the no-penetration condition on the cavity end wall, an infinite-dimensional matrix equation for the duct mode coefficients is obtained. However, for a fixed frequency, only a finite number of the duct modes are cut-on. The rest decay exponentially away from their point of origin, so that only a finite number of modes are important at the location of the end wall. The infinite-dimensional matrix equation can then be truncated at finite order and solved numerically. The rapid convergence of the solutions with increasing matrix order is remarkable. In fact, the procedure works well even in the limit of a rectangular cavity (when the overhang length b is set to zero), and the exponential decay of the higher-order terms is replaced by algebraic decay.

The evolution of each component of the unsteady field in propagating from one end of the cavity to the other is then calculated using the method of multiple scales to account for the streamwise variation of the shear layer. The scattering and propagation transfer functions are then combined in the form of a homogeneous matrix equation $AX = 0$, where $X = [\mathcal{U}, \mathcal{S}, \mathcal{D}, \mathcal{E}_f, \mathcal{E}_d]^T$. The complex frequencies of eigenfrequencies are determined by the condition $\det(A) = 0$. Since the eigenfrequencies are complex, the analysis determines both the temporal frequency and the growth (or decay) rate of each mode.

Theoretical predictions have been carried out for both supersonic and subsonic flows. Comparisons with experimental results for both supersonic and subsonic flows are very encouraging. Our predictions for the temporal frequency are generally in good agreement with experimental data; in some cases the agreement with experimental data is better than that obtained with Rossiter's semi-empirical formula. We believe this is a significant accomplishment, considering the fact that our theory contains no empirical constants. Our theory also determines whether each mode is unstable or stable; this important aspect of the physics was not addressed by Rossiter's semi-empirical formula or previous theoretical models.

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References

[1] Rossiter J.E.: Wind Tunnel Experiments on the Flow Over Rectangular Cavities at Subsonic and Transonic Speeds. Royal Aircraft Establishment, TR No. 64307, October 1964.

[2] Heller H.H., Bliss D.B.: The Physical Mechanism of Flow Induced Pressure Fluctuations in Cavities and Concepts for Suppression. AIAA Paper 75-491, March 1975.

[3] Tam C.K.W., Block P.J.: On the Tones and Pressure Oscillations Induced by Flow over Rectangular Cavities. *J. Fluid Mech* **89**:373–399, 1978.

[4] Noble B.: Methods Based on the Wiener-Hopf Technique. Chelsea 1988.