GLOBAL STABILITY OF THE FLOW INDUCED BY WALL INJECTION

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Summary The paper deals with the study of the stability of the plane flow induced by wall injection. The perturbation is in the normal mode form in time and the eigenfunction is described in the (x, y)-plane, where the flow also takes place. This approach allows a better comprehension of the thrust oscillations observed in large solid propellant motors.

PRESENTATION

Large solid propellant motors exhibit oscillations of their thrust certainly due to the coupling of the acoustics and a hydrodynamic instability of the flow inside. In this paper, we only focus on the study of the hydrodynamic instability of the flow. In order to simplify the analysis, compressibility and 3D effects are neglected. The flow is induced by lateral wall injection of fluid, with a constant velocity, in a semi-infinite plane channel.

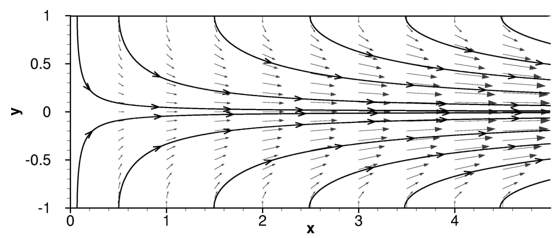

Figure 1. Streamlines and velocity field of the flow induced by wall injection.

The mean flow can be described by a streamfunction $\Psi(x, y)$ whose expression is $\Psi(x, y) = x \sin\left(\frac{\pi}{2} y\right)$ according to the Culick/Taylor relation valid for large Reynolds numbers. The coordinate x is the longitudinal one, with $x \in [0, \infty[$, and y the transverse one, $y \in [-1, 1]$, see figure 1. Both are made dimensionless by half the height of the channel. This mean flow is strongly non-parallel.
The purpose of the present paper is the study of the global behaviour in the (x, y)-plane of any perturbation superimposed to the Taylor/Culick flow. A streamfunction Φ associated with the perturbation is sought in the normal mode form but only in time t, so that Φ can be written as:

$$\Phi(x, y, t) = \phi(x, y) e^{-i\omega t},\ \omega \in \mathbb{C}.$$

The function ϕ depends on x and y, what is the only acceptable form considering that the mean flow is non-parallel. Classical "quasi-parallel" stability analysis have been previously performed (see [1], [2]), but they are not valid for small values of x in our case (see figure 1).
The linearised Navier-Stokes equations write as a partial differential equation for ϕ with respect to the two variables x and y

$$-\frac{\partial\Psi}{\partial x}\frac{\partial\Delta\phi}{\partial y} - \frac{\partial^3\Psi}{\partial y^3}\frac{\partial\phi}{\partial x} + \frac{\partial\Psi}{\partial y}\frac{\partial\Delta\phi}{\partial x} + \frac{\partial^3\Psi}{\partial x\partial y^2}\frac{\partial\phi}{\partial y} - \frac{1}{R}\Delta\Delta\phi = i\omega\Delta\phi,$$

with Δ the Laplacian operator $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ and R the Reynolds number based on half the height and the wall injection velocity. The boundary conditions associated with the previous equation are homogeneous

- At the lateral wall: $\phi(x, \pm 1) = 0$ and $\frac{\partial\phi}{\partial y}(x, \pm 1) = 0$, for all $x \geq 0$,
- At the head of the channel: $\phi(0, y) = 0$ and $\frac{\partial\phi}{\partial x}(0, \pm 1) = 0$, for all $y \in [-1, 1]$.
- An "adequate" output condition for ϕ

what leads to an eigenvalues problem with ω the complex unknown eigenvalue. For its numerical resolution, it is then possible to separate symmetric and antisymmetric solutions in order to divide by two the size of the computational domain. Results are given in the lower half plane $(x, y) \in [0, X_f] \times [-1, 0]$, with X_f the final abscissa of the computational domain. It seems that the "adequate" boundary condition imposed at the exit has nearly no effect on the results.

RESULTS

Two kinds of results can be obtained, first the spectrum of the eigenvalue problem and second the eigenfunctions associated with it. The spectrum gives the value of the pulsation (ω_r, real part of ω) and also the temporal growth rate of the intrinsic perturbation (ω_i, imaginary part). An example (antisymmetric mode for $R = 1000$ and $X_f = 4$) is given figure 2 in the plane (ω_r, ω_i). It can be observed that the spectrum is discrete, so only particular frequencies correspond to intrinsic perturbation of the Taylor/Culick flow. Then, a coupling of the acoustics with the hydrodynamic perturbations is possible when frequencies corresponding to an acoustic mode (a cavity mode) and to a perturbation are the same. It may be a simple way to understand the creation of the thrust oscillations. It can also be observed that the temporal growth rate ω_i of the perturbations is negative, it means that they are vanishing with time t, so a constant source of noise is necessary in order to observe the perturbations. This seems to agree with direct resolution of Navier-Stokes equations in this case: every one hundred time-steps, noise must be injected at the lateral wall for the oscillations to be observed. In experimental set-up, the noise is provided “naturally”. The temporal growth rate ω_i is also decreasing with an increase of the pulsation, this is certainly due to the viscosity of the fluid, the more the phenomenon is varying rapidly, the more it is damped by the viscosity.

If we look at the phase of ϕ, figure 3, a special care must be taken for particular points of these functions. Except on the boundary of the computational domain, the perturbation of the stream-function may present some points Ax ($x = 1, 2, \ldots$, see figure 3) with a modulus equal to zero. For these particular points, the phase of the wave is not defined, and there exist discontinuity lines $\mathcal{L}x$ of the argument of the complex function ϕ, defined in $]-\pi, \pi]$ for figure 3. The lines $\mathcal{L}x$ link the points Ax to a boundary. To each line $\mathcal{L}x$ corresponds a discontinuity line of the phase of Φ which turns in time around each point Ax. These particular points are comparable to amphidromic points when studying the tide of the seas where the amplitude of the tide is always zero-value, and the tide is turning around these points. They are due to the effect of the Coriolis force that curves the stream of the seas and then is responsible for the large amplitude of the tides, much more than the effect of the moon or of the sun that mainly impose the period of the tide (half a day).

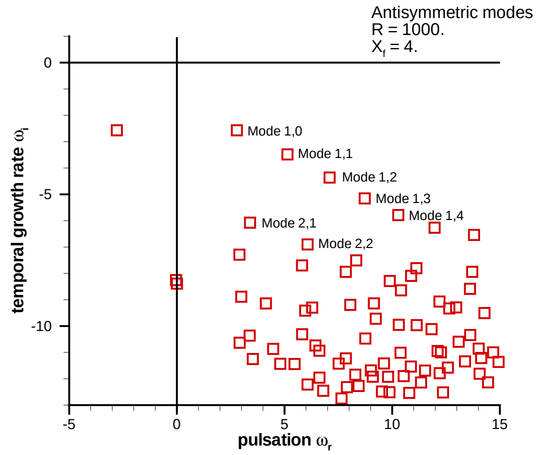

Figure 2. Part of the spectrum of the Taylor/Culick flow for the antisymmetric modes.

By having a look at the first numerically converged modes, enumerated in figure 2, it is possible to classify the modes with the number of amphidromic points of the eigenfunction associated with. Each mode j, i has i amphidromic points. Moreover, except for the mode $i = 0$ whose amplitude is decreasing with x, each mode exhibits a quasi-exponential increase of the amplitude of the function ϕ in the x-direction, see last graphic in figure 3 which represents the real part of ϕ in the (x, y)-plane for the mode 1,2. The increase of the perturbation may be due to the curvature of the streamlines that may act as the Coriolis force on the tide.

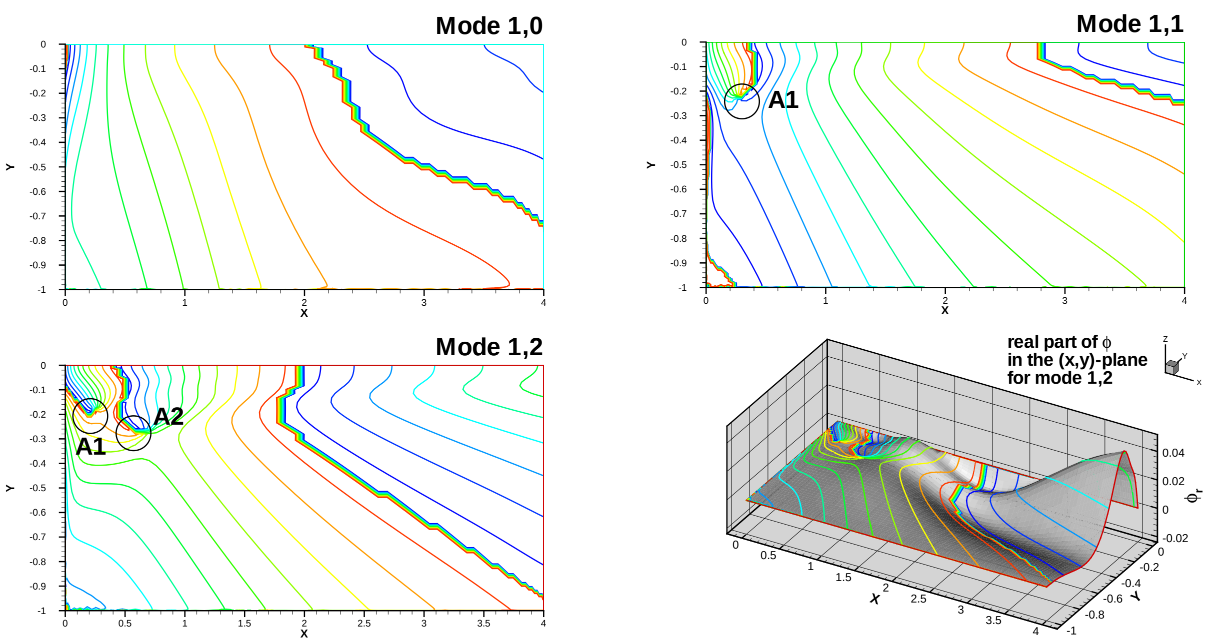

Figure 3. Phase of the eigenfunctions ϕ for modes (1,0), (1,1), (1,2). The number of amphidromic points, Ai, depends on the frequency of the perturbation. The last graphic represents ϕ_r that is oscillating and growing in x.

References

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