ACOUSTIC FIELD GENERATED BY INSTABILITY WAVES IN THE TRANSONIC REGIME

Christophe Millet*, Stéphane Le Dizès**
*LDG, CEA, BP 12, 91680 Bruyères-le-Châtel, France
**IRPHE, CNRS-UMR 6594, 49 rue Joliot-Curie, BP 146, 13384 Marseille cedex 13, France

Summary We consider the problem of acoustic radiation generated by a spatial instability wave on a weakly-developing two-dimensional mixing layer. Assuming a WKBJ approximation for the instability wave, we compute the far pressure field by using a Fourier transform along the streamwise direction. Approximations for this pressure field are obtained by a steepest descent method when the WKBJ parameter goes to zero. A branch cut and several saddle points are shown possibly to contribute to the approximation. A detailed analysis of these contributions is provided when the instability wave is close to transonic near its maximum amplitude. This permits to explain the modifications of the far pressure field observed during a subsonic-supersonic transition.

INTRODUCTION

The processes by which instability waves generate acoustic radiation have been discussed in the past 20 years (see, for example, Tam [3], Goldstein [2] for review articles, Crighton and Huerre [1] for an asymptotic study of superdirective acoustic sources). Using the wavy wall analogy, it was suggested that the direction of the most intense noise radiation from a supersonic jet or mixing layer can be estimated by using the Mach angle relation based on the speed of the most amplified instability wave. However, Tam and Burton [4, 5] pointed out that the analogy must be modified to account for the growth and decay of the instability wave as it propagates downstream. They demonstrated that the far field should be modified near the location where the instability wave starts to decay but they do not try to fully analyse this transition. For subsonic flows, the noise characteristics are known to be different than their supersonic counterparts as no such transition exists. Yet, the formal solution given by Tam and Burton still applies. The goal of this work is to analyse this solution when we are close to the transonic regime and near the amplitude maximum in order to understand how the supersonic transition occurs.

FAR FIELD IN THE WKBJ FRAMEWORK

We consider a linear compressible instability wave of real frequency ω on a developing two-dimensional mixing layer. In the mixing layer, the perturbation pressure amplitude is assumed to be described, near its maximum, by a WKBJ approximation of the form

$$p_i(X,y) \sim p_o(X,y) \exp\left\{\frac{i}{\epsilon}\int^X \alpha\right\} \tag{1}$$

where y is the transverse coordinate and α is the local (complex) wavenumber which depends on the slow streamwise variable $X = \epsilon x$, characteristic of the evolution length of the mixing layer.
Following Tam and Burton [4], a leading order expression of the pressure far field can be obtained by Fourier transform in terms of the outer variables (X, Y), with $Y = \epsilon y$, as

$$p_f(X,Y) \sim c_\epsilon \int_{-\infty}^{+\infty} q(K) \exp\left\{\frac{\psi(K,X,Y)}{\epsilon}\right\} \mathrm{dK}\,, \tag{2}$$

where c_ϵ is an amplitude factor, $q(K)$ a function which is given by Tam and Burton [4] but whose explicit expression is not important here, and

$$\psi(K,X,Y) = i\int^{\overline{X}} \alpha(s)\,\mathrm{ds} - iK(\overline{X} - X) - \lambda(K)Y. \tag{3}$$

In (3), $\overline{X}$ is a function of K defined by $\alpha(\overline{X}) = K$. The function λ can be written, with a suitable choice of non-dimensionalization as

$$\lambda(K) = \sqrt{K^2 - 1}\,. \tag{4}$$

In order to be able to evaluate the function $\overline{X}$ near the branch points $K = \pm 1$ in a general setting, we consider a spatial location X close to the maximum amplitude location X_m defined by $\alpha_i(X_m) = 0$, where α_i is the imaginary part of α, and assume that $\alpha(X_m)$ is also close to 1. This assumption means that we consider an instability wave which is transonic at its maximum. In that way, it can be shown that the phase ψ reduces near $K = 1$ to a "generic" expression of the following form

$$\psi(K,X,Y) \sim \psi(1,X,Y) + i(K-1)(\tilde{X} + a) - b(K-1)^2 + \sqrt{2(K-1)}Y \tag{5}$$

where a is a coefficient proportional to $\alpha(X_m) - 1$, b a coefficient connected to $\alpha'(X_m)$ and $\tilde{X} = X - X_m$.
This "generic" expression permits to understand how the instability field, which is recovered for small Y is transformed into a pure acoustic field for large $\tilde{X}$ and Y.

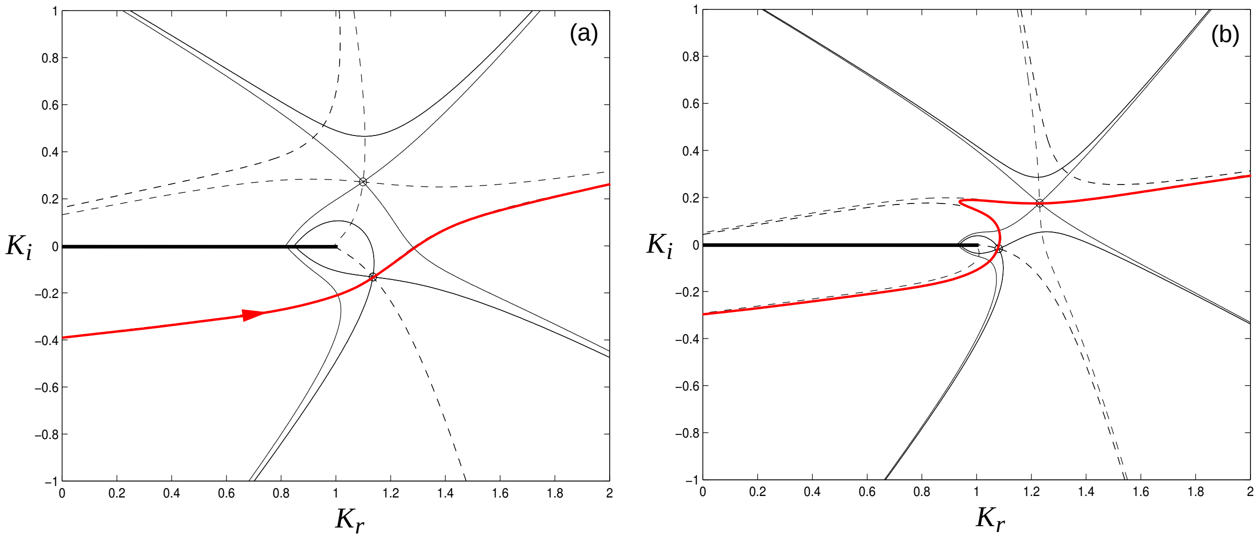

Figure 1. Locus of saddle points (circles) and branch cut (thick black line) in the complex K-plane ($K = K_r + iK_i$). Thin solid lines and thin dashed lines are constant levels of real and imaginary parts of $\psi(K)$, respectively. The thick red line gives the integration contour. Control parameters are given by $a = -i$, $b = e^{-i\pi/8}$ at the location defined by $\tilde{X} = 0$ and $Y = 0.56$ for (a) or $Y = 0.35$ for (b). For $Y \to 0$, it is possible to show that the saddle-point contribution of (a) degenerates into a branch point contribution. The saddle-point of the upper complex half-plane gives the instability wave.

An asymptotic expression of the far pressure field is obtained by the steepest descent method in the limit of small ϵ. The integration contour is deformed in the complex plane in order to go through all the relevant saddle points. Two saddle points are shown to intervene in the analysis, together with the branch point singularity. According the parameters a and b, and the values of the variables $\tilde{X}$ and Y, one or two contributions may be present. In figure 1 are displayed examples of phase contours in the complex K-plane for which there exist one (a) and two (b) saddle-point contributions. The different contributions may also change dominance as the variables are modified. This occurs along particular curves in the $(\tilde{X}, y)$ plane.

When the instability wave is supersonic at its maximum X_m, the main contribution downstream X_m, is found to be associated with the branch point even for Y as small as $O(\sqrt{\epsilon})$. This means that, in some regions, the acoustic field could become dominant over the instability wave that has generated it. It is expected to generate new instability waves in the mixing layer region, downstream X_m. This feedback phenomenon could affect the perturbation field in the mixing layer.

CONCLUSIONS

In this study, we have derived far field approximations for the perturbation field generated by a WKBJ instability wave in a compressible two-dimensional mixing layer. Using a local analysis near the location of perturbation maximum amplitude, we have been able to determine the generic structure of the acoustic far field close to the transonic regime. This has permitted to give a complete description of the subsonic-supersonic transition. It has also demonstrated that the supersonic damped wave continuation described by Tam and Burton [4] is not sufficient to provide the complete structure of pressure fluctuations in the whole physical space. In particular, we have shown that a dominant branch point contribution remains present in the perturbation field downstream the maximum.

Note finally that the analysis, provided here for a mixing layer, can be extended in a straightforward manner to treat the case of axisymmetric jets.

References

[1] Crighton D. G. and Huerre P. : Shear-layer pressure fluctuations and superdirective acoustic sources *J. Fluid Mech.* **220**:355–368, 1990.

[2] Goldstein M. E. : Aeroacoustics of turbulent jet flows. *Ann. Rev. Fluid Mech.* **16**:263–95, 1984.

[3] Tam C. K. W. : Supersonic jet noise. *Ann. Rev. Fluid Mech.* **27**:17–43, 1995.

[4] Tam C. K. W. and Burton D. E. : Sound generated by instability waves of supersonic flows. Part 1. Two dimensional mixing layers. *J. Fluid Mech.* **138**:249–271, 1984.

[5] Tam C. K. W. and Burton D. E. : Sound generated by instability waves of supersonic flows. Part 2. Axisymmetric jets. *J. Fluid Mech.* **138**:273–295, 1984.