THERMO-MECHANICAL STABILITY AND VIBRATION ANALYSIS OF COMPOSITE SHELLS

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The postbuckling and postbuckled vibration analysis of curved panels subjected to thermo-mechanical loading is presented here. The formulation is based on the modified Sanders' theory incorporating the geometric nonlinearities. The multi-term Galerkin's technique is used to obtain the true postbuckled shape of curved panels, postbuckled frequencies and associated mode shapes.

The buckling and postbuckling behavior of laminated curved panels (spherical and cylindrical) are investigated extensively and reported in the literature. However, relatively less work has been done in the area of postbuckled free vibration analysis of laminated curved panels under thermo-mechanical loading. Librescu et al. [1-3] presented the analytical results of simply supported single-layer and three-layer flat and curved panels made from transversely isotropic materials. The authors solved the nonlinear boundary-value problem using Airy's stress function and one-term Galerkin's approximation. Most of the analytical results presented in the literature are based on the Donnell approximation. The main advantage of the Donnell approach is the possibility of using Airy stress function F, where the number of unknown functions is reduced from three (u, v, w) to two (w, F). The validity of the w-F formulation vis-à-vis its u-v-w counterpart has been investigated by Sheinman and Goldfeld [4] and showed that the w-F fails to bring out the lowest buckling load accurately.

The postbuckling and postbuckled vibration results of curved panels subjected to thermo-mechanical loading are presented here. The formulation is based on the modified Sanders' theory incorporating the geometric nonlinearities. The higher-order shear deformation displacement field used in the present study is that of Reddy and Liu [5], which accounts for parabolic distribution of the transverse shear strains through thickness of the shell and tangential stress-free boundary conditions on the boundary surfaces of the shell.

Adopting Galerkin's procedure, the governing nonlinear partial differential equations are converted into a set of nonlinear algebraic equations in the case of postbuckling analysis and nonlinear ordinary differential equations in the case of free vibration analysis. The critical buckling temperature is obtained from the solution of linear eigenvalue problem. The postbuckled equilibrium paths are obtained by solving the nonlinear algebraic equations, using the Newton-Raphson iterative procedure. In the case of free vibration analysis of buckled panel, the solution of differential equations is assumed to be the sum of time dependent solution (vibration amplitude) and time independent solution (postbuckling deflection). The vibration amplitude is considered small compared to the postbuckled deflection and hence higher order time dependent terms are neglected. The free vibration frequencies of a postbuckled shell panel about the static equilibrium state are obtained by solving the eigenvalue problem for different postbuckled deflections. Numerical results are presented for curved composite panels with and without initial geometric imperfections. The modal participation of each mode in the postbuckling deflection is obtained using multi-term Galerkin's procedure. The results show the thermo-mechanical load interaction on buckling, limiting points and snap-through buckling. Apart from the fundamental mode, higher vibration modes of curved panels about the prebuckling and postbuckling equilibrium states subjected to thermo-mechanical loading are presented.

Response of doubly curved shell panels subjected to thermo-mechanical loads: The nonlinear equilibrium paths of doubly curved symmetric [*0/90/0*] cross-ply shell panels under combined uniform temperature distribution (T in ^{o}C) and sinusoidal distributed lateral load are presented in Fig 1. The simply supported immovable edge boundary conditions are considered at all edges. The curves are plotted showing the variation of nondimensional load parameter q^* with nondimensional central deflection (w/h). Fig. 2 shows the fundamental frequencies of a doubly curved shell corresponding to the static equilibrium paths shown in the Fig 1 where, ω_o represents the fundamental frequency of unloaded shell. The imaginary frequencies below the abscissa shown in the figure 2 correspond to unstable equilibrium paths shown in the Fig.1.

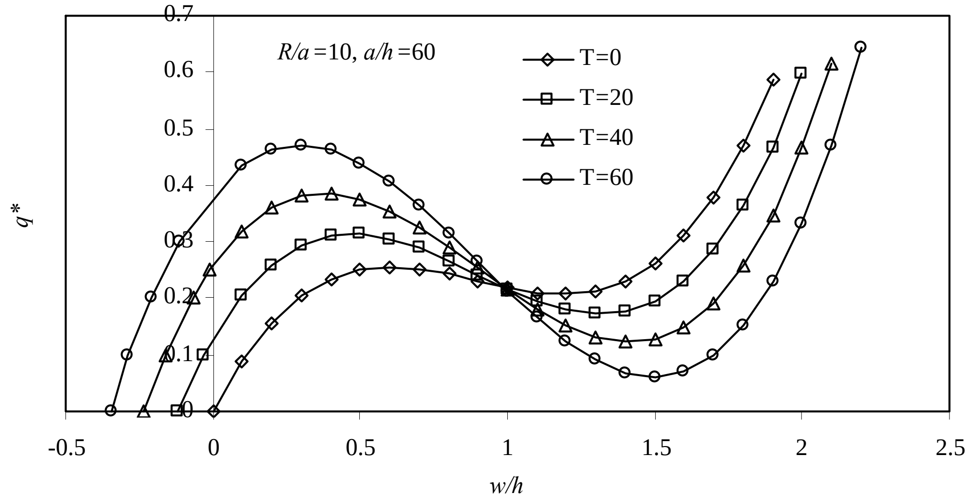

Figure 1: Response of doubly curved shell panel [*0/90/0*] subjected to thermo mechanical loads. ($a/b = 1$, $R_1 = R_2 = R$, $q^* = q_0 R_2/E_2 h^2$, $E_1 = 25E_2$, $G_{12} = G_{13} = 0.5E_2$, $G_{23} = 0.2E_2$, $\nu_{12} = 0.25$, $\alpha_2/\alpha_1 = 3$)

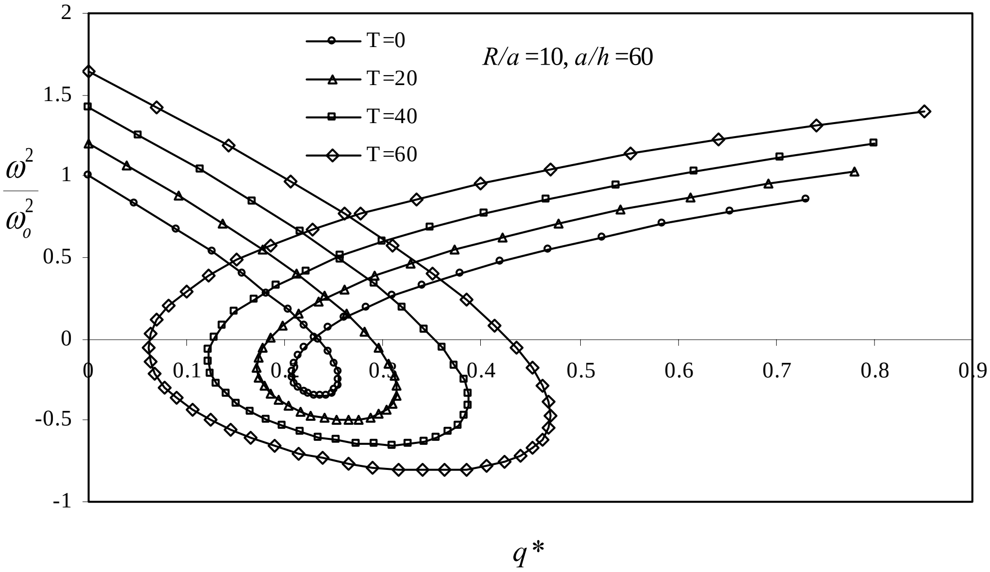

Figure 2: The influence of uniform temperature distribution on the fundamental frequencies of symmetric cross-ply spherical shell panel under sinusoidal distributed load.

References:

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