## SELF-EXCITED STICK-SLIP OSCILLATIONS OF DRAG BITS

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#### INTRODUCTION

Rotary drilling systems used to drill deep boreholes for hydrocarbon exploration and production often experience severe torsional vibrations, called stick-slip, characterized by (i) sticking phases where the bit stops, and (ii) slipping phases where the angular velocity of the tool  $\Omega$  increases up to two times the imposed angular velocity. This problem is particularly acute with drag bits, which consist of fixed blades or cutters mounted on the surface of a bit body. The main attributes of observed stick-slip oscillations are:

Modelling of the stick-slip oscillations of drag bits is typically carried out by considering only the torsional vibration of the drill string and by reducing the bit-rock interface to an equivalent frictional contact with a velocity weakening friction coefficient. In this paper, we propose an alternative approach based on a discrete model of the drilling system and on a rate-independent description of the bit-rock interaction. This model takes into consideration the axial and torsional vibration modes and their coupling through bit-rock interaction laws, which account for both frictional contact and cutting processes at the bit-rock interface. We show that this model experiences self-excited vibrations, which can degenerate into stick-slip oscillations or bit bouncing under certain conditions.

### MATHEMATICAL MODEL

A rotary drilling structure consists essentially of a rig, a drill string, and a bit. The principal components of the drill string are the bottom hole assembly (BHA) composed mainly of heavy steel tubes to provide a large downward force on the bit, and a set of drill pipes made of thinner tubes. For the idealized drilling system under consideration, we assume that a constant upward force  $H_0$  and a constant angular velocity  $\Omega_0$  are applied by the rig on the drill string, that the borehole is vertical, and that there are no spurious lateral motions of the bit.

The bit motion as well as the forces acting on the bit can in principle be calculated knowing the prescribed surface boundary conditions, the mechanical properties of the drill string and the bit-rock interaction law. For this analysis, we consider a discrete model of the drill string stripped to its essential elements, i.e., a point mass M and moment of inertia I to represent the BHA and a linear spring of torsional stiffness C to model the drillpipes, see Figure 1.

Given the constant angular velocity  $\Omega_o$  and constant upward tension  $H_o$  imposed to the upper end of the torsional spring, we seek to determine the unknown bit response  $\mathcal{R}$ . This response, which is generally a function of time t, consists of two sets of conjugated dynamic and kinematic quantities: the weight-on-bit W and the vertical bit velocity V on the one hand, and the torque-on-bit T and the angular bit velocity  $\Omega$  on the other hand. We introduce the vertical position U and the angular position  $\Phi$  of the bit. There is a steady-state (trivial) response of the bit, denoted as  $\mathcal{R}_o = \{W_o, V_o, T_o, \Omega_o\}$ , which is characterized by constant quantities  $W_o = W_s - H_o$ ,  $\Omega = \Omega_o$ ,  $\Phi = \Omega_o t - T_o/C$  where  $W_s$  is the submerged weight of the drillstring. The non-trivial response  $\mathcal{R}$  can be determined in principle from the surface conditions and bitrock interface laws, together with the following torsional and axial equations of motion

$$I\frac{d^{2}\Phi}{dt^{2}} + C(\Phi - \Phi_{o}) = T_{o} - T, M\frac{d^{2}U}{dt^{2}} = W_{o} - W$$

We restrict consideration to an idealized drag bit of radius a, consisting of n identical radial blades regularly spaced by an angle equal to  $2\pi/n$ , see Figure 1. Each blade is characterized by a sub-vertical cutting surface and a wearflat of constant width  $\ell_n$  orthogonal to the bit axis. The depth of cut per blade  $d_n(t)$  is constant along the blade and identical for each blade;  $d_n$  is given by

$$d_n(t) = U(t) - U(t - t_n)$$

where  $t_n(t)$  is the time required for the bit to rotate by an angle  $2\pi/n$  to its current position at time t. The delay  $t_n(t)$  is solution of

$$\Phi(t) - \Phi(t - t_n) = 2\pi / n$$

The nature of the bit-rock interaction depends on the bit axial and angular velocity.

Consider first the "normal" case when V > 0 and  $\Omega > 0$ . The drilling action of a drag bit consists of a pure cutting process in front of each blade and a frictional process along the wear flats. Both the weight-on-bit W and the torque-on-bit T can thus be expressed as  $W = W_c + W_f$ ,  $T = T_c + T_f$  where the subscript c denotes the cutting component, and f the frictional component. The forces associated with the cutting process are taken to be proportional to the depth of cut d while the frictional forces mobilized along the wearflats depend on a rate-independent friction coefficient  $\mu$ . The cutting and frictional components of W and T are explicitly given by (Detournay & Defourny 1992)

$$W_c = a\zeta \varepsilon d$$
,  $W_f = a\ell \sigma$ ,  $T_c = \frac{a^2}{2}\varepsilon d$ ,  $T_f = \frac{a^2}{2}\eta\mu\ell\sigma$ 

where  $\varepsilon$  is the intrinsic specific energy (the amount of energy required to cut a unit volume of rock),  $\zeta$  characterizes the inclination of the cutting force on the cutting face (typically,  $0.6 \le \zeta \le 0.8$ ), and  $\sigma$  is the magnitude of the normal stress acting across the wear flat interface (single cutter experiments show that  $\sigma \sim \varepsilon$ ). The number  $\gamma$  globally characterizes the spatial orientation and distribution of the chamfers/wearflats. When the bit is moving upwards, we assume a complete loss of contact between the wearflat and the rock ( $W_f = T_f = 0$ ).

The case  $V = \Omega = 0$  corresponds to the stick phase when the bit remains immobile. Since rotation of the drill pipes continues at the surface, the torque applied by the drillstring on to the BHA builds up until its magnitude is sufficient to overcome the reacting torque; the bit slips.

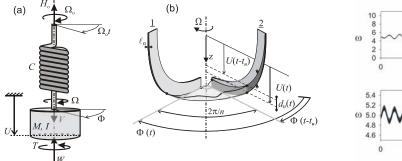
It is convenient to formulate the model in a dimensionless form via a characteristic length  $L_*$  and time  $t_*$ . The scaled bit response depends on two sets of parameters: the control parameters  $\mathcal{W}_0 = W/\zeta \varepsilon a L_*$  and  $\omega_0 = \Omega_0$   $t_*$ ; and the problem parameters characterizing the geometry and wear state of the bit, the rock, and the drilling structure, i.e., the number of blades n, and the lumped parameters  $\beta = \mu \gamma \zeta$ ,  $\lambda = n\sigma \ell/\zeta \varepsilon L_*$ , and  $\Psi = \zeta \varepsilon a I/MC$ .

# SELF-EXCITED OSCILLATIONS

A linear stability analysis indicates that the trivial motion is unstable. Numerical solution of the system of equations confirms that any perturbations to the trivial motion cause the system to evolve towards one regime of self-excited axial and torsional vibrations. Several such regimes exist, with some characterized by stick-slip oscillations or bit bouncing. It appears that most of these regimes of solutions are either periodic characterized by a limit cycle or quasi-periodic, i.e. they evolve extremely slowly compared to the characteristic time of the system. Figure 2 illustrates the evolution of the bit angular velocity  $\omega$  towards a limit cycle for two cases corresponding to  $\beta$ =0.3 and  $\beta$ =1 (lower and upper plot, repsectively).

The root cause of the self-excitation is the cutting process which introduces, via the delayed axial position of the bit, a feedback into the equations of motion. Growth of the axial oscillations is hindered, however, by an intermittent "high-frequency" loss of contact at the wearflat/rock interface, which is limiting the transfer of energy from the torsional to the axial motion. The coefficient  $\beta$ , which encapsulates the asymmetry between the axial and torsional interface laws, affects the ratio between the energy going into the cutting process and the energy dissipated in frictional contact. In fact, stick-slip oscillations take place only if  $\beta$ <1

The results of an extensive series of numerical experiments can be summarized into the following general observations. In the absence of bit bouncing, the system reaches strictly a limit cycle whenever  $\beta$ >1, or if stick-slip develops when  $\beta$ <1, see Figure 2. If there is no stick-slip and bit bouncing in the case  $\beta$ <1, the response reaches "rapidly" a quasi-periodic regime, with the amplitude of the torsional oscillations increasing extremely slowly, presumably towards a limit cycle with stick-slip oscillations. The susceptibility to stick-slip motion increases with a reduction of the angular velocity  $\omega_0$  and with an increase of  $W_0$ , in accordance with field observations.



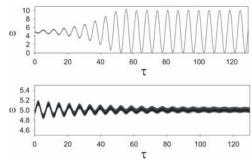


Figure 1: (a) Drillstring model. (b) Section of bottom-hole

Figure 2: Angular velocity vs time (dimensionless)

### **CONCLUSIONS**

The proposed model differs in significant ways from the standard approach used to analyze stick-slip torsional vibrations. We consider both axial and torsional vibrations of the bit, as well as the coupling between the two vibration modes through the bit-rock interaction laws. Second, the interface laws account both for cutting of the rock and for frictional contact between the cutter wearflats and the rock. Within the framework of the discrete model considered here in this paper, the evolution of the system is governed by two coupled delay differential equations, with the delay being part of the solution, and by discontinuous contact conditions. The proposed model qualitatively reproduces all the field observations (rate dependence of the mean torque, influence of the operating parameters  $W_0$  and  $\omega_0$  on the susceptibility to stick-slip).

### References

[1] Detournay, E and P. Defourny (1992) A phenomenological model of the drilling action of drag bits. Int. J. Rock Mech. Min. Sci & Geomech. Abstr. 29 (1), 13-23