

# GENERATING OPTIMAL MOTIONS OF CONSTRAINED MULTIBODY SYSTEMS

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*Summary* A general approach to optimal motion synthesis of constrained multibody systems is presented. It applies to controlled mechanical systems such as industrial manipulators and legged-locomotion systems. An optimal control problem is stated. A parametric optimization technique based on approximating joint motion coordinates using spline functions of class  $C^3$  is developed to recast this primary problem into an optimization problem of mathematical programming. The latter is solved using SQP algorithms.

## INTRODUCTION

Fully actuated multibody systems such as robotic manipulators and walking machines present kinematic redundancies which require organizing their movements appropriately. Furthermore, we are especially interested with motion phases during which the mechanical system may present kinematic loops. In that case, over-actuation must be solved suitably in order to avoid undesirable stress in the closed kinematic chains, together with antagonistic forces which could result in sudden break of contact, or sliding when one-sided contacts are at stake. In any case, there is the need for generating well coordinated movements, dynamically efficient and few energy consuming. A general approach consists in setting and solving an optimization problem that minimizes a dynamic performance criterion. In this way, a constrained optimal control problem is stated. It can be solved using two main computational techniques. In [1], Pontryagin's Maximum Principle was implemented to generate a variety of optimal motions. In this presentation, we are interested in more widespread techniques based on parametric optimization as in [2-4]. Such an approach helps to cope with the computational complexity of dynamic models of multibody systems. Moreover, the optimization problem can be solved in that case using sequential quadratic programming algorithms which are computationally quite efficient.

## AN OPTIMAL CONTROL PROBLEM

### Dynamic modeling

We consider any rooted multibody systems with tree-like topology and having possibly kinematic loops when moving. All joints are assumed to be actuated. The common approach used for modeling the constrained dynamics of such systems consists in considering closed loops as cut at appropriate joints or contacts, while accounting for relevant closure conditions which restore the original kinematics. Then, introducing a full set of configuration coordinates put together in the  $n_q$ -vector  $\mathbf{q}$ , a Lagrangian dynamic model can be formulated as the differential-algebraic set of equations

$$t \in [0, T], \begin{cases} \mathbf{M}(\mathbf{q}(t))\ddot{\mathbf{q}}(t) + \mathbf{C}(\mathbf{q}(t), \dot{\mathbf{q}}(t)) + \mathbf{G}(\mathbf{q}(t)) = \mathbf{A}_\tau(\mathbf{q}(t))\boldsymbol{\tau}(t) + \boldsymbol{\Phi}^T(\mathbf{q}(t))\boldsymbol{\lambda}(t) \\ \mathbf{C}_c(\mathbf{q}(t)) = 0 \end{cases} \quad (1)$$

$$(2)$$

The  $n_c$ -order vector-function  $\mathbf{C}_c$  represents a set of closure constraints. Its Jacobian matrix is  $\boldsymbol{\Phi}(\mathbf{q}) \equiv \partial \mathbf{C}_c / \partial \mathbf{q}$ . In (1),  $\mathbf{M}$  stands for the mass-matrix, the vector function  $\mathbf{C}$  contains centrifugal and Coriolis inertia terms, and  $\mathbf{G}$  represents gravity terms. The  $n_\tau$ -vector  $\boldsymbol{\tau}$  stands for actuating joint torques, and  $\boldsymbol{\lambda}$  is the  $n_c$ -vector of Lagrange's multipliers associated with closure constraints  $\mathbf{C}_c$ .  $\mathbf{A}_\tau$  is a  $(n_\tau \times n_q)$ -matrix depending on the choice of generalized coordinates.

### Constraints defining feasible movements

Some boundary conditions must be accounted for as initial and final equality constraints:

$$\mathbf{C}_0(\mathbf{q}(0), \dot{\mathbf{q}}(0)) = \mathbf{0} \in \mathfrak{R}^{n_0}, n_0 \leq n_q; \mathbf{C}_T(\mathbf{q}(T), \dot{\mathbf{q}}(T)) = \mathbf{0} \in \mathfrak{R}^{n_T}, n_T \leq n_q \quad (3)$$

Joint motion and velocity limitations, actuating torques limitations, obstacle avoidance, and one-sided contact conditions, result in constraints we represent formally as the set of inequalities:

$$t \in [0, T], k \leq n_h, h_k(\mathbf{q}(t), \dot{\mathbf{q}}(t), \boldsymbol{\tau}(t), \boldsymbol{\lambda}(t)) \leq 0 \quad (4)$$

Some  $h_k$  in (4) do not depend explicitly on all variables introduced in (4).

### Performance criterion

The performance criterion we want to minimize is basically the integral amount of quadratic joint actuating torques. However, as  $\boldsymbol{\lambda}$  in (1) represents a set of forces to be applied in order to hold the tips of cut kinematic chains in their prescribed positions defined by the closure constraints (2), we consider such efforts as additional active forces. In this way, we add a quadratic function in  $\boldsymbol{\lambda}$  to the Lagrangian of a quadratic functional in  $\boldsymbol{\tau}$  to be minimized, namely:

$$\text{Minimize } J, J(\boldsymbol{\tau}, \boldsymbol{\lambda}) = \frac{1}{2} \int_0^T [\boldsymbol{\tau}(t)^T \boldsymbol{\tau}(t) + \boldsymbol{\lambda}(t)^T \boldsymbol{\lambda}(t)] dt \quad (5)$$

Then, the problem to be solved consists in computing a triple-vector time-function  $t \rightarrow (\mathbf{q}(t), \boldsymbol{\tau}(t), \boldsymbol{\lambda}(t))$  minimizing the criterion  $J$  in (5) while satisfying the conditions (1) to (4). Let us underline that the minimization of  $\boldsymbol{\lambda}$  through  $J$ , plays a key role in our approach for succeeding in generating optimal motions of constrained multibody systems

## STATING A PARAMETRIC OPTIMIZATION PROBLEM

A key point in dealing with parametric optimization applied to motion synthesis lies in the representation of time functions such as generalized coordinates, using a finite set of discrete parameters. In [2-3], coefficients of polynomial functions are dealt with as optimization parameters for approximating generalized coordinates. A more accurate approach consists in fitting spline-functions at knots uniformly distributed along the motion time. In [4], splines of class  $C^2$  were used for optimizing gait cycles of a planar biped. However, it appeared that optimal actuating torques could have jerky variations at knots. This situation may be overcome using splines of class  $C^3$  ensuring the differentiability of joint accelerations at knots. We define such functions as the concatenation of 4-order polynomials as follows:

$$\{t_1 (= 0), t_2, \dots, t_k, \dots, t_N (= T)\}; \forall k \leq N-1, t_{k+1} - t_k = \alpha > 0, t \in [t_k, t_{k+1}], \begin{cases} \tau = (t - t_k)/(t_{k+1} - t_k) \\ q_i(t) \equiv \varphi_{ik}^*(\tau) = \sum_{j=0}^4 c_{ikj} \tau^j \end{cases} \quad (6)$$

Connecting polynomials  $\varphi_{ik}^*$ 's at knots up to their third derivative yields spline functions of class  $C^3$ . Then considering  $x_i$ 's in

$$\mathbf{x} = (x_1, \dots, x_{n_x})^T \equiv (q_1(t_1), \dots, q_1(t_N), \dots, q_{n_q}(t_1), \dots, q_{n_q}(t_N)); \dot{q}_i(0), \dot{q}_i(T), i \leq n_q)^T$$

as a set of optimization parameters, the connecting conditions of  $\varphi_{ik}^*$ 's result in the linear system:  $\mathbf{M}_c \mathbf{c} = \mathbf{M}_x \mathbf{x}$  in which  $\mathbf{c}$  stands for a column-vector of coefficients  $c_{ijk}$ 's, and  $\mathbf{M}_c$  is a sparse, hexadiagonal matrix. Assuming that  $\mathbf{M}_c$  is invertible, the vector  $\mathbf{c}$  can be expressed as the linear function of  $\mathbf{x}$ :  $\mathbf{c} = \mathbf{M}_c^{-1} \mathbf{M}_x \mathbf{x}$ . Thus, generalized coordinates  $q_i$ 's in (8) can be written as functions of  $\mathbf{x}$  and  $t$ , namely

$$\mathbf{x} \in \mathcal{R}^{n_x}, i \leq n_q, q_i(t) \equiv \varphi_i(\mathbf{x}, t) \quad (7)$$

Then, the next step consists in expressing  $\boldsymbol{\tau}$  and  $\boldsymbol{\lambda}$  as functions of  $(\mathbf{x}, t)$  through (7) and (1). Defining the matrix  $\mathbf{A} = (\mathbf{A}_\tau, \boldsymbol{\Phi}^T)$  and the vector  $\mathbf{u}^T = (\boldsymbol{\tau}^T, \boldsymbol{\lambda}^T)$ , equation (1) may be rewritten using (7), as the vector equation linear in  $\mathbf{u}$ :

$$t \in [0, T], \mathbf{A}(\mathbf{x}, t) \mathbf{u}(t) = \mathbf{b}(\mathbf{x}, t) \quad (8)$$

If  $\boldsymbol{\lambda} \neq \mathbf{0}$ ,  $\mathbf{A}$  is not invertible, and equation (8) is underdetermined in  $\mathbf{u}$ . However,  $\mathbf{A}$  is a full rank matrix. Thus for any given  $(\mathbf{x}, t)$ , it is possible to extract a minimum norm solution using the right pseudo-inverse matrix  $\mathbf{A}^+$  of  $\mathbf{A}$ , namely

$$\mathbf{u}(t) \equiv \mathbf{u}^*(\mathbf{x}, t) \equiv (\boldsymbol{\tau}^*(\mathbf{x}, t)^T, \boldsymbol{\lambda}^*(\mathbf{x}, t)^T)^T = \mathbf{A}^+(\mathbf{x}, t) \mathbf{b}(\mathbf{x}, t) \quad (9)$$

Using this new expression for  $\mathbf{u}$ , the criterion (5) is changed into the real function of  $\mathbf{x}$

$$F(\mathbf{x}) = \frac{1}{2} \int_0^T [\boldsymbol{\tau}^*(\mathbf{x}, t)^T \boldsymbol{\tau}^*(\mathbf{x}, t) + \boldsymbol{\lambda}^*(\mathbf{x}, t)^T \boldsymbol{\lambda}^*(\mathbf{x}, t)] dt \quad (10)$$

Now, through (7) and (9), we account for closure constraints (2), boundary conditions (3), and constraints (4) as the double set of equality and inequality constraints defined at every knots  $t_k$ :

$$k \leq N, \begin{cases} j \leq N^e, C_j^e(\mathbf{x}, t_k) = 0 \\ j \leq N^i, C_j^i(\mathbf{x}, t_k) \leq 0 \end{cases} \quad (11)$$

The initial optimal control problem defined by the statements (1) to (5) is then restated through (10) and (11) as a constrained, non-linear minimization problem of mathematical programming. It can be efficiently solved using computing codes available in numerical software libraries.

## CURRENTLY AVAILABLE SIMULATIONS and CONCLUSIONS

The above approach was implemented to generate optimized cyclic steps of a human-like seven-link planar biped. The only data required to create a gait pattern is the walking velocity. Step length, postural configurations at transitions between single-support and double-support phases, and transition times between successive phases are optimized as well. It was possible to generate smooth gait cycles for any walking speed between 0.4m/s and 1.3m/s. Quite restrictive constraints such as unilaterality of ground-foot contacts and non-sliding conditions are perfectly satisfied.

The method presented yields suboptimal solutions having smooth properties: no undesirable oscillations for the  $q_i$ 's, and no jerks for the  $\tau_i$ 's. It does not require a great number of control points. It could be easily developed for industrial and space manipulators, as well as to simulate human movements.

## References

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