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NONLINEAR TRANSITION OF A FLOW DRIVEN BY A ROTATING MAGNETIC FIELD

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Summary Results of direct numerical simulation are presented confirming that a linearly stable liquid metal flow driven by a rotating magnetic magnetic field can experience transition to turbulence. This is possible due to a non-normal nonlinear mechanism which governs a transition in several generic shear flows. The employed numerical approach can be used for nonlinear instability analysis of various flows.

INTRODUCTION

Linearly stable shear flows may become turbulent due to finite disturbances. The theoretical explanation of this phenomenon named 'non-normal nonlinear transition' [1] has two ingredients. If the eigenvectors of a linearized system are not orthogonal (non-normal), then some directions in phase space may be very poorly spanned. Consequently, the amplitude of 'misfit' disturbances may grow considerably before getting aligned with the eigendirections. Thus, an initially small initial disturbance may eventually 'catapult' the flow into a nonlinear regime governed by additional subcritical solutions of the Navier–Stokes equations. Such solutions need not to be connected with the basic flow at any finite value of the control parameter. Although unstable [2], these solutions may trap the flow temporarily, at least. It may happen, for instance, if the time-scale of approaching the additional unstable steady state is much less than the characteristic escape time. Besides, the stable manifolds of additional steady states may densely fill the phase space, thus disabling straight return paths. As a result, the disturbance lifetime may be essentially unpredictable and finite as evidenced by direct numerical simulation of the plane Couette flow [2].

The flow due to rotating magnetic field (RMF) is known to have alternative steady solutions [4] very close to the linearly stable basic solution. Experiments [5] have shown that tiny imperfections can cause an irregular onset well below the linear instability threshold. Thus, the linear analysis alone is not sufficient to understand transition of this particular flow. Is this flow so particular or may similar effects take place also in other flows beyond the classical examples? We demonstrate at the example of an RMF driven flow in a cylindrical cavity that the non-normal nonlinear transition can be predicted by direct numerical simulation of the flow response to random forcing.

PROBLEM FORMULATION

The rotating magnetic field is considered in the commonly employed approximation when the skin-effect is neglected and the electromagnetic driving force is flow independent. In this approximation the RMF induces a steady body force with an analytical expression given in [4]. The magnitude of this force in dimensionless terms is the so-called magnetic Taylor number

$$Ta = \frac{\sigma\omega_o B_o^2 H_o^4}{\rho\nu^2},$$

where ω_o and B_o is frequency and the magnetic flux density of the rotating field; σ, ρ and ν is electric conductivity, density and kinematic viscosity of the liquid metal, respectively, but H_o is half-height of the cylinder (assumed equal to the radius in this study). The flow is described by the Navier-Stokes equations with fixed azimuthal body force and by the divergence free constraint. No-slip conditions are applied at the cylinder surface. To simulate noise a random body force is added.

METHOD

The three-dimensional solution $\mathbf{v}$ was decomposed into normal azimuthal modes. Each of these modes was expressed in Chebyshev series of radial and axial coordinates. Symmetries of smooth functions with respect to the radial coordinate were taken into account so that only appropriate parity (even or odd) radial modes were included [6]. The Navier–Stokes equations in primitive variables were integrated in time by a second order implicit scheme. The pressure boundary conditions were evaluated by the so-called spectral dependency matrix. Any of the Chebyshev expansion coefficients of the random body force (representing the noise) were set to a zero mean Gaussian distributed random variable in each time step. The flow response depends on the spatial and temporal resolution in this approximation of noise. Therefore, the timestep and number of decomposition modes were kept constant in all runs. The flow response was expressed in terms of the disturbance energy $\varepsilon = 0.5\int_V (\mathbf{v}-\mathbf{v}_o)^2 dV$, where $\mathbf{v}_o$ is the basic steady solution. The noise was normalized and scaled by an amplitude factor A in such a way that the disturbance energy $E = 1/T\int_0^T \varepsilon(t)dt = A^2$ for zero basic flow. The integration time was $T = 0.5$ in this study which corresponds to about 50 to 100 revolutions of the core flow depending on *Ta*. The same sequence of random forcing was applied in each run.

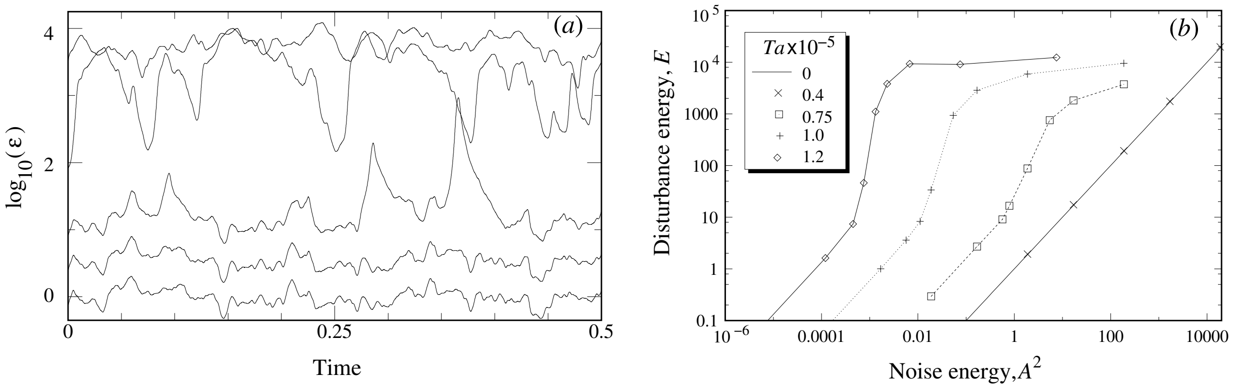

Figure 1. Transient disturbance energy at a variable noise amplitude, $Ta = 10^5$ (a) and the response function (b).

RESULTS

The azimuthal body force drives an almost rigidly rotating core flow [3] separated from the end walls by the so-called Bödewadt layers. Angular momentum decreases outwards in the side layer supporting a Taylor–Görtler instability there. The meridional flow caused by Ekman pumping, however, instantly deforms and washes away growing Taylor vortices. Thus, the first linear onset at $Ta_c = 1.232 \times 10^5$ is different from the Taylor–Görtler instability [6]. Additional unstable solutions in form of Taylor vortices bifurcate somewhere well above the linear instability limit and continue down to $Ta_g = 0.366 \times 10^5$ [5].

We considered the flow at several fixed values of the forcing parameter $Ta = 0.4, 0.75, 1.0, 1.2 \times 10^5$ from the metastability range $Ta_g < Ta < Ta_c$ and observed four regimes. At the lowest forcing the flow response showed random scatter and did not differ considerably from the response of a motionless state. The energy of response increased at a higher forcing and the time dependency of the response got smoother. The response energy increased linearly with the energy of noise $O(A^2)$ in this second regime. After reaching a certain noise amplitude the flow response experienced a qualitative change characterized by an intermittent nonlinear growth of the disturbance energy. The duration of nonlinear outbursts increased until they merged in a continuous series in the fourth regime (Fig. 1a). All four regimes can be recognized in the $E(A^2)$ dependency as shown in Fig. 1b. The average response energy scaled with the noise energy $E = kA^2$ with $k = \text{const} \geq 1$ in the first two (linear) regimes. The response grew much faster than $O(A^2)$ in the third regime, and, finally, it was almost insensitive to the noise amplitude in the fourth regime.

DISCUSSION AND CONCLUSIONS

The observations above are in line with two main general features of the non-normal nonlinear transition mechanism [1]. Namely, small disturbances are considerably amplified due to the non-normality of the linearized operator and big enough disturbances are further amplified (temporarily) by nonlinear interactions. The lifetime of the nonlinear structures is often the central issue concerning the nonlinear transition. In our case these nonlinear structures are simply the Taylor vortices born in couples and advected by the basic meridional flow. As soon as the vortices leave the boundary layer they decay linearly. Although the nonlinear structures are not self-sustaining in the RMF flow, their growth is faster than the following linear decay. Consequently, if we consider an instant noise or repeating disturbances, the lifetime of nonlinear structures may only influence the steepness of the response function $E(A^2)$ in the beginning of the nonlinear regime. Assuming that noise is a more appropriate model of natural imperfection than an isolated disturbance, we conclude that a cumulative disturbance amplification factor is practically a more important issue than the lifetime of a single excited state. Note that this factor is always finite as far as we consider the linearly stable flow. In case of the RMF flow this factor can reach values of about 10^6 which explains why hardly visible imperfections could promote onset in the experiment [5].

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