XXI ICTAM, 15–21 August 2004, Warsaw, Poland

BOUNDARY LAYERS INDUCED BY CONTACT OF ROUGH BODIES

Stanisław Stupkiewicz*
*Institute of Fundamental Technological Research, Świętokrzyska 21, 00–049 Warsaw, Poland

Summary A micromechanical framework is developed for the analysis of deformation inhomogeneities in the boundary layers that are induced by contact of rough bodies. The main idea is to average the inhomogeneous fields along the contact surface but preserve the dependence of the averages on the distance from the surface. Some properties of such averages are provided. As an example a boundary layer associated with ploughing by an array of periodic sine-shaped asperities is analyzed by the finite element method.

INTRODUCTION

In the case of contact of rough bodies, the characteristic lengths of the roughness are typically much smaller than those of the bodies. Thus two points of view can be adopted. At the *micro-level* the stress transfer is concentrated at small spots, so called real contacts, and the distribution of the contact pressure is highly inhomogeneous. These inhomogeneities govern the deformation of surface asperities and their interaction. At the *macro-level* it is the slowly-varying average (macroscopic) contact pressure that determines the overall deformation of the contacting bodies.

Friction laws are typically formulated in terms of the normal and tangential components of the contact traction vector $\boldsymbol{t} = \boldsymbol{\sigma} \cdot \boldsymbol{n}$, where $\boldsymbol{\sigma}$ is the stress tensor at the contact surface and $\boldsymbol{n}$ is the unit surface normal. Consequently, only the *exterior* part [1] of the stress tensor $\boldsymbol{\sigma}$ is involved in the description and the complete stress/strain state in the vicinity of the contact surface is typically not accounted for. However, in some situations, the *interior* (in-plane) parts of stress or strain significantly affect friction and other contact phenomena, particularly when the deformation in the sub-surface layer is inhomogeneous. This is for example the case of metal forming processes where the surface asperities are flattened more easily in the presence of bulk plastic deformation. This leads to high real contact area fractions [2, 3], even at moderate contact pressures. Depending on the lubrication regime this can result in an increased adhesive friction component or affect lubrication conditions. A closely related effect is also observed in hardness indentation testing, where the in-plane stresses affect the force-penetration response [4].

It seems that the effects associated with deformation inhomogeneities in the sub-surface contact layers and the interaction of these inhomogeneities with the macroscopic stresses and strains have not attracted sufficient attention in the literature yet. The aim of this work is thus to develop a micromechanical framework that would allow for consistent analysis of these effects and, in a broader perspective, would help to develop improved constitutive laws of contact phenomena.

BOUNDARY LAYERS: AVERAGING, COMPATIBILITY CONDITIONS AND PROPERTIES

Consider a homogeneous body occupying domain Ω with *micro-inhomogeneous* boundary conditions $\boldsymbol{t}^*$, the traction, and $\boldsymbol{u}^*$, the displacement, prescribed on the parts of boundary $\partial_t\Omega$ and $\partial_u\Omega$, respectively. By micro-inhomogeneity of $\boldsymbol{t}^*$ ($\boldsymbol{u}^*$) we understand that it consists of a slowly varying, average field $\bar{\boldsymbol{t}}^*$ ($\bar{\boldsymbol{u}}^*$) and its fluctuation $\tilde{\boldsymbol{t}}^*$ ($\tilde{\boldsymbol{u}}^*$). The wave-length of the fluctuation field is assumed small compared to the dimensions of Ω. It is also assumed that the inhomogeneity of deformation induced by the inhomogeneous boundary conditions is confined to a thin sub-surface layer, the *boundary layer*, along $\partial_t\Omega$ and $\partial_u\Omega$.

The equations of the boundary layer are obtained using the method of asymptotic expansions of the classical homogenization [5], under the assumption of periodicity of fluctuation fields $\tilde{\boldsymbol{t}}^*$ and $\tilde{\boldsymbol{u}}^*$. As a result, in addition to the *macroscopic* b.v.p. in Ω with *micro-homogeneous* boundary conditions $\bar{\boldsymbol{t}}^*(\boldsymbol{x})$ on $\partial_t\Omega$ and $\bar{\boldsymbol{u}}^*(\boldsymbol{x})$ on $\partial_u\Omega$, the *local* b.v.p. of the boundary layer is obtained for each point along the boundaries $\partial_t\Omega$ and $\partial_u\Omega$. The local b.v.p. of the boundary layer is a problem of a homogeneous half-space with a periodic traction or a periodic displacement prescribed along its boundary (the conditions of frictional contact can be treated accordingly). The displacement field in the boundary layer has the form

$$\boldsymbol{u}(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{u}_0(\boldsymbol{x}) + \boldsymbol{E}(\boldsymbol{x})\cdot\boldsymbol{y} + \boldsymbol{w}(\boldsymbol{y}), \qquad \boldsymbol{w}(\boldsymbol{y}^+) = \boldsymbol{w}(\boldsymbol{y}^-), \qquad \boldsymbol{w}(\boldsymbol{y}) \stackrel{y_3\to\infty}{\longrightarrow} \boldsymbol{0}, \tag{1}$$

where $\boldsymbol{y} = \boldsymbol{x}/\epsilon$, $\epsilon \ll 1$, is the local coordinate, $\boldsymbol{E}(\boldsymbol{x})$ is the macroscopic strain at point $\boldsymbol{x}$ on the boundary $\partial_t\Omega$ or $\partial_u\Omega$ and $\boldsymbol{w}$ is the unknown additional displacement field, which is periodic within the plane (S-periodic) and vanishes far from the plane (the local coordinate system is adopted such that the y_3-axis is normal to the surface).

The averaging operation is then defined. For a field $\varphi(\boldsymbol{y})$ its average $\bar{\varphi}$ at fixed y_3 is given by

$$\bar{\varphi}(y_3) = \langle\varphi\rangle \equiv \frac{1}{|S|}\int_S \varphi(\boldsymbol{y})\,\mathrm{d}y_1\mathrm{d}y_2, \qquad |S| = \int_S \mathrm{d}y_1\mathrm{d}y_2, \tag{2}$$

where S is the periodicity cell within the (y_1, y_2)-plane. The inhomogeneous field $\varphi(\boldsymbol{y})$ can now be decomposed into its average value $\bar{\varphi}(y_3)$ and fluctuation $\tilde{\varphi}(\boldsymbol{y})$, so that $\varphi(\boldsymbol{y}) = \bar{\varphi}(y_3) + \tilde{\varphi}(\boldsymbol{y})$ and $\langle\tilde{\varphi}\rangle = 0$.

Several properties of stress and strain averages can now be derived. The derivation is omitted here and three most important properties are provided below. First of all, the following compatibility conditions hold:

$$\Delta\bar{\boldsymbol{\sigma}}_A \equiv \bar{\boldsymbol{\sigma}}_A - \boldsymbol{\Sigma}_A = \boldsymbol{0}, \qquad \Delta\bar{\boldsymbol{\varepsilon}}_P \equiv \bar{\boldsymbol{\varepsilon}}_P - \boldsymbol{E}_P = \boldsymbol{0}, \tag{3}$$

where subscripts A and P denote, respectively, the exterior and the interior parts of a symmetric tensor [1], so that for example $\boldsymbol{\sigma} = \boldsymbol{\sigma}_A + \boldsymbol{\sigma}_P$ and $\boldsymbol{\sigma}_P \cdot \boldsymbol{n} = \boldsymbol{0}$. The macroscopic stress (i.e. the uniform stress far from the surface) is denoted by $\boldsymbol{\Sigma}$ and the macroscopic strain by $\boldsymbol{E}$.
Secondly, the (double) average total strain energy density is given by

$$\langle \boldsymbol{\sigma} : \boldsymbol{\varepsilon} \rangle = \bar{\boldsymbol{\sigma}} : \bar{\boldsymbol{\varepsilon}} + \frac{\mathrm{d}}{\mathrm{d}y_3} \langle \boldsymbol{n} \cdot \tilde{\boldsymbol{\sigma}} \cdot \tilde{\boldsymbol{w}} \rangle \,, \tag{4}$$

where $\boldsymbol{\sigma}$ is a statically admissible field, $\boldsymbol{\varepsilon}$ is the strain derived from a kinematically admissible displacement field of the form (1) with $\boldsymbol{w} = \bar{\boldsymbol{w}} + \tilde{\boldsymbol{w}}$. Note that the Hill's postulate of the classical micromechanics of heterogeneous materials (i.e. $\{\boldsymbol{\sigma} : \boldsymbol{\varepsilon}\} = \{\boldsymbol{\sigma}\} : \{\boldsymbol{\varepsilon}\}$, where $\{\cdot\}$ is the average over the representative volume) is not satisfied in the case of inhomogeneous boundary layers.
Consider now an elastic-plastic body. The local constitutive relation is $\boldsymbol{\sigma} = \boldsymbol{L} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p)$ with a constant elastic moduli tensor $\boldsymbol{L}$ (the inelastic strain $\boldsymbol{\varepsilon}^p$ is, in general, inhomogeneous in the boundary layer). An analogous relation holds for the averages, $\bar{\boldsymbol{\sigma}} = \boldsymbol{L} : (\bar{\boldsymbol{\varepsilon}} - \bar{\boldsymbol{\varepsilon}}^p)$, while far from the surface we have $\boldsymbol{\Sigma} = \boldsymbol{L} : (\boldsymbol{E} - \boldsymbol{E}^p)$. In view of compatibility conditions (3) the average stress $\bar{\boldsymbol{\sigma}}(y_3)$ is fully determined if $\bar{\boldsymbol{\varepsilon}}^p(y_3)$ is known, namely

$$\Delta\bar{\boldsymbol{\sigma}}(y_3) = -\boldsymbol{Q}\Delta\bar{\boldsymbol{\varepsilon}}^p(y_3), \qquad \Delta\bar{\boldsymbol{\sigma}}(y_3) = \bar{\boldsymbol{\sigma}}(y_3) - \boldsymbol{\Sigma}, \qquad \Delta\bar{\boldsymbol{\varepsilon}}^p(y_3) = \bar{\boldsymbol{\varepsilon}}^p(y_3) - \boldsymbol{E}^p, \tag{5}$$

where the operator $\boldsymbol{Q}$ depends on $\boldsymbol{L}$ and $\boldsymbol{n}$. Similar relations hold also for the average strain $\bar{\boldsymbol{\varepsilon}}(y_3)$. Note that Eqn. $(5)_1$ is a special case of the interfacial relations that hold for local stress/strain jumps at regular discontinuities [1].

EXAMPLE: ASPERITY PLOUGHING

As an example consider a two-dimensional (plain strain) asperity ploughing problem. An array of rigid sine-shaped asperities ploughs through an elastic-plastic half-space which is subjected to macroscopic in-plane tensile or compressive strain $E_{11} = E_{11}^0$. After the sliding distance of $2l$ (l being the asperity spacing) the surfaces are separated and the macroscopic in-plane strain is released, $E_{11} = 0$. The finite element mesh of a periodic cell of boundary layer is shown in Fig. 1(a), where the distribution of plastic multiplier during relative sliding is also shown. It is seen that the plastic deformation is localized close to the real contact and thus inhomogeneous. The distribution of the average stress $\bar{\sigma}_{11}(y_3)$ normalized by the yield stress σ_y is shown in Fig. 1(b,c). The effect of the macroscopic in-plane strain E_{11}^0 on the residual stress $\bar{\sigma}_{11}$ in the boundary layer after separation and release of the in-plane strain is clearly visible in Fig. 1(c).
Note that it is only because of the inhomogeneity of deformation that plastic deformations appear in the sub-surface layer. Application of an equivalent (in terms of the average value) uniform normal and tangential traction would result in a purely elastic response (as it is the case far from the surface where the inhomogeneities vanish). Also note that the interior part of the average stress $\bar{\boldsymbol{\sigma}}_P$ (e.g. $\bar{\sigma}_{11}$) deviates from its macroscopic counterpart only in a part of the boundary layer adjacent to the surface, namely in the zone of non-zero plastic deformations, cf. Fig. 1(b,c), in agreement with relations (5).

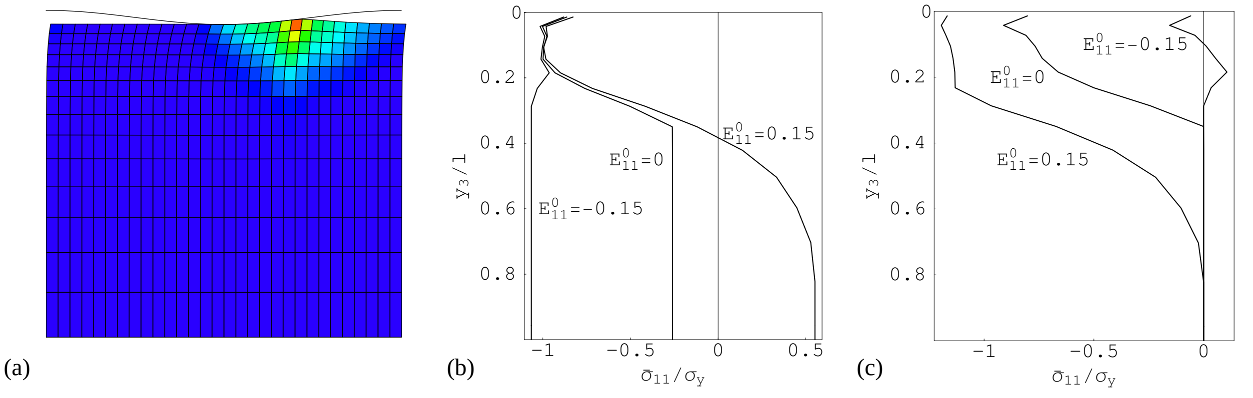

Figure 1. Asperity ploughing: (a) distribution of plastic multiplier; (b) distribution of the normalized average stress $\bar{\sigma}_{11}(y_3)/\sigma_y$, with applied in-plane strain $E_{11} = E_{11}^0$ and (c) after the in-plane strain is released, $E_{11} = 0$.

Acknowledgement. This work has been supported by the EC, ENLUB project, contract No. G1RD-CT-2002-00740.

References

[1] Hill R.: Interfacial operators in the mechanics of composite media. *J. Mech. Phys. Solids* **31**:347–357, 1983.
[2] Wilson W.R.D., Sheu S.: Real area of contact and boundary friction in metal forming. *Int. J. Mech. Sci.* **30**:475–489, 1988.
[3] Stupkiewicz S., Mróz Z.: Phenomenological model of real contact area evolution with account for bulk plastic deformation in metal forming. *Int. J. Plast.* **19**:323–344, 2003.
[4] Giannakopoulos A.E.: The influence of initial elastic surface stresses on instrumented sharp indentation. *ASME J. Appl. Mech.* **70**:638–643, 2003.
[5] Sanchez-Palencia. E.: Boundary layers and edge effects in composites. In E. Sanchez-Palencia and A. Zaoui, editors, *Homogenization Techniques for Composite Media, Lecture Notes in Physics* **272**:121–192, Springer, Berlin, 1987.