DISTURBED-LAMINAR FLOW OVER AN OSCILLATING CYLINDER

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Summary The flow around a cylinder oscillating at small amplitude in fluid otherwise at rest is subject to instabilities that, at a certain amplitude, trigger a regime of three-dimensional disturbed-laminar flow. This paper discusses apparent conflicts that are evident in recent observations, and describes new measurements of hydrodynamic damping and flow visualisations. Attention is directed at the range 1,000 < β < 20,000 where β is the Stokes parameter based on the cylinder's diameter.

The flow around a circular cylinder oscillating in a direction normal to its axis with a small amplitude-to-diameter ratio in a fluid otherwise at rest is relevant in several practical applications and represents a fundamental problem of longstanding interest to fluid dynamicists. A reasonable starting point is to assume that the flow is laminar and two-dimensional, and for this case Stokes (1851) obtained a celebrated solution which was later extended by Wang (1968). A key result is that the fluid is predicted to resist the motion of the cylinder with a force

$$W = 2\pi\mu U\left[(\pi\beta)^{\frac{1}{2}} + 1 - \frac{1}{4}(\pi\beta)^{-\frac{1}{2}} + \ldots\right], \quad (1)$$

per unit length, where $\mu = \nu\rho$ is the viscosity, U is the cylinder's velocity, and β is the Stokes parameter $d^2 f/\nu$; d is the diameter of the cylinder and $f = \omega/2\pi$ its frequency. There is also a viscous force that is in phase with the acceleration, but this is dominated by the inertia force and is generally of less practical importance.

In connection with the problem of predicting the hydrodynamic damping of compliant offshore structures, recent efforts have concentrated on measuring the force on a cylinder oscillating at high β and small Keulegan Carpenter number, $K = 2\pi a/d$ where a is its amplitude. The results of these measurements have helped to identify the range of validity of equation (1). In 1981 Honji discovered a now well-known instability which marks a boundary between 'laminar' and 'disturbed-laminar' regimes, and one upper limit for the validity of the Stokes-Wang solution. In disturbed-laminar flow the fluid motion remains well-organised but has three-dimensional features not present in the laminar regime. An example from our measurements is shown in figure 1.

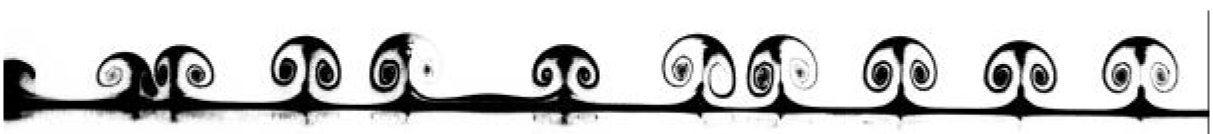

Figure 1. 'Mushroom' vortices along the widest part of the cylinder generated by oscillatory motion in the direction normal to the page.

A stability analysis by Hall (1984), valid for $\beta \to \infty$, indicated that the two-dimensional flow would first become unstable to these Taylor-Görtler vortices when the Keulegan Carpenter number of the motion exceeded

$$K_{\mathrm{H}} = \frac{5.78}{\beta^{1/4}}\left(1 + \frac{0.21}{\beta^{1/4}} + \ldots\right). \quad (2)$$

This agrees with observations of the conditions in which three-dimensional flow first occurred, as can be seen in figure 2, which includes Honji's results and other data from of Sarpkaya (1986). Both these and Honji's results seem to confirm K_{H} as the threshold for the onset of the disturbed-laminar regime. Turbulence and separation first appeared at amplitudes that were greater than this by about another 50%.

However, more recent measurements of forces at higher values of β suggest that in other conditions equation (2) does not represent the limit of validity of the Stokes-Wang theory. Vertical lines in figure 2 identify the ranges over which forces have been measured, in experiments using a smooth cylinder oscillating at small amplitudes in still water – or in the kinematically identical case of a stationary cylinder in an oscillating flow. Each one is labelled with the observed amplitude of the damping force, in terms of the Stokes-Wang result W, defined in equation (1). These experiments become more and more difficult as β is increased, but techniques have evolved considerably over the last 10 years and there is now widespread agreement (supported also by further measurements of Sarpkaya at $\beta \approx 10^6$ (2001)) on a rather surprising result. For values of β from some threshold to the highest values yet reached, and for Keulegan Carpenter numbers that extend as much as *two orders of magnitude below* K_H, the damping is very close to $2W$. This suggests that the flow is subject to some other instability that has not yet been identified, since it seems very unlikely that small disturbances described by Sarpkaya (2002) can alone be responsible for a doubling of the viscous force. Recent measurements by Johanning (2003) indicate that at constant K the jump from W to $2W$ takes place at around $\beta = 4{,}500$.

The aims of the work described in this paper are to review the current position and attempt an answer to the question: why does the damping force on an oscillating cylinder in fluid otherwise at rest not appear to agree with the Stokes-Wang theory, when $K < K_H$ and β is greater than some threshold of order 10^3? The paper presents new measurements and visualisations of the flow in the region where the changes take place and in the light of this, discusses the apparent conflicts in earlier measurements of hydrodynamic damping.

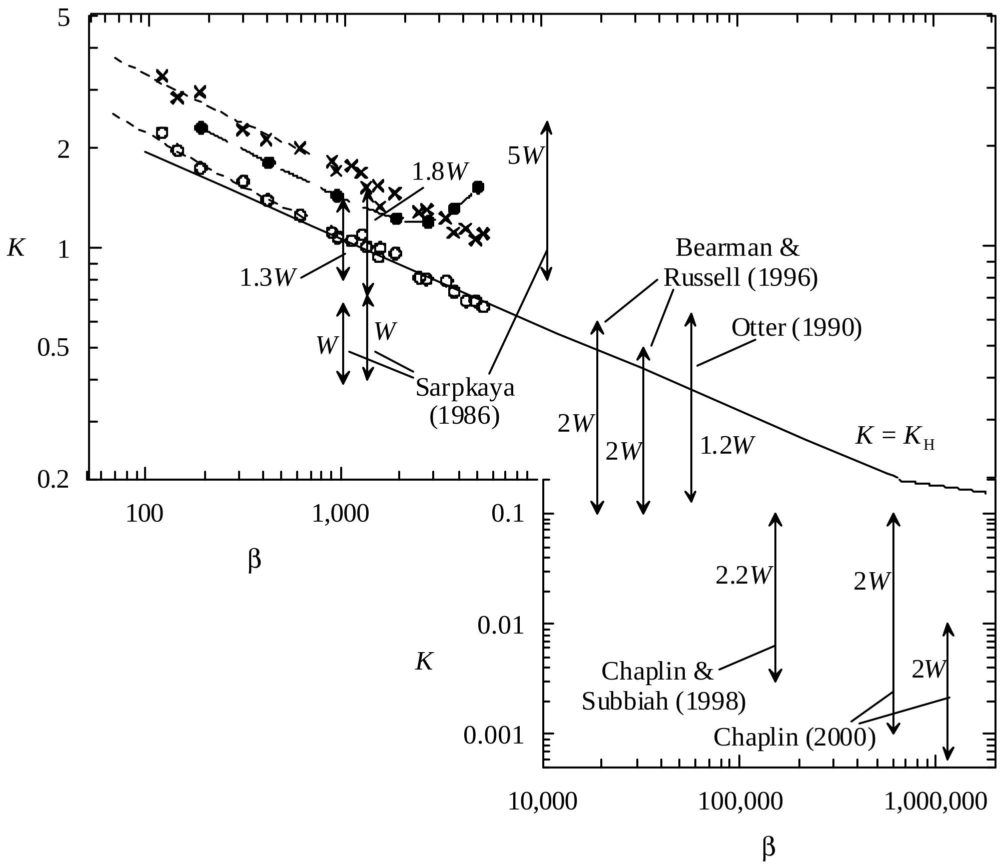

Figure 2. Measurements of flow and forces on a cylinder oscillating at small amplitudes in water otherwise at rest. Apart from the broken lines (which pass through Honji's (1981) data) all the results for $\beta < 12{,}000$ are from Sarpkaya (1986) and indicate observations of the onset of the Honji instability (○); separation (●); turbulence (×). Vertical arrows represent the ranges over which force measurements have been made, with an adjacent indication of the force magnitude, in terms of the Stokes-Wang result W. Hall's stability criterion (2) is shown as a continuous line.

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