GENERALISED BEAM THEORY FORMULATION TO ANALYSE THE POST-BUCKLING BEHAVIOUR OF ORTHOTROPIC LAMINATED PLATE THIN-WALLED MEMBERS

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ABSTRACT

This paper presents the derivation and illustrates the application of a non-linear orthotropic GBT formulation, which is intended to perform post-buckling analyses of laminated plate FRP open-section thin-walled members. Different types of loading and end support conditions can be dealt with and the theory can handle the presence of arbitrary initial geometrical imperfections. One is able to determine “exact” and “approximate” (only a few modes) post-buckling equilibrium paths and the evolution, along those paths, of (i) displacements and stresses and, using the GBT unique mode decomposition feature, also of (ii) the deformation mode participation in the member deformed configuration. To validate and illustrate the application and capabilities of the formulated GBT, numerical results, concerning the post-buckling behaviour of laminated plate FRP lipped channel members exhibiting different orthotropic behaviours, are presented and discussed. Some of them are compared with values obtained from finite element analyses, performed in the code ABAQUS and adopting shell element meshes to discretise the member.

INTRODUCTION

The structural use of *composite materials* started in the 1950’s and has steadily progressed since then, mainly due to developments taking place in the aeronautical industry, which have led to a wide range of products extensively and routinely employed at present. However, the picture has been quite different in civil engineering, as composite material applications became significant only in the last few years, when their well known (i) structural efficiency (low weight/strength and weight/stiffness ratios) and (ii) excellent behaviour under aggressive environmental conditions were matched by (iii) sufficiently low fabrication costs. In particular, the combination of these three features is responsible for the growing demand for *thin-walled* composite structural members recently observed in the construction industry, namely related to off-shore structures and chemical plants.

As it would seem logical to expect, the heterogeneity and orthotropic constitutive relations of the composite materials render their mechanical behaviour considerably more complex than the one displayed by metals such as steel or aluminium, thus introducing additional difficulties to the analysis of thin-walled structural members. In particular, most composite materials exhibit often (i) linear elastic stress-strain relationships (with relatively low moduli), (ii) no ductility (*i.e.*, an elastic behaviour up to collapse) and (iii) different types of orthotropy (depending on the material constituents and fabrication procedure), leading to mechanical properties that clearly indicate a high susceptibility to (i) local and/or global *instability phenomena* (see figure 1) and (ii) brittle collapse modes. Since mastering these two aspects is of paramount importance to achieve safe and economical (competitive) designs, engineers must be equipped with analytical/numerical tools that are able to model accurately their influence on the structural behaviour and load-carrying capacity of thin-walled composite members. This requires not only the access to specific and sophisticated methods of analysis but also an “unbiased mind”, in the sense that is necessary to be suspicious of intuition-based reasonings developed in the context of isotropic materials.

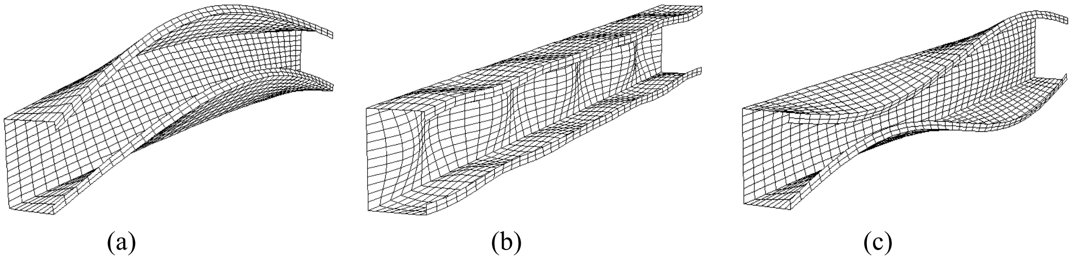
(a) (b) (c)

Fig. 1. Thin-walled member flexural-torsional, local-plate and distortional buckling

A few years ago, a rather elegant and quite powerful theory to analyse the structural behaviour of isotropic elastic prismatic thin-walled members was developed by Schardt [1] and designated as *Generalised Beam Theory* (GBT). This theory incorporates both local (cross-section) and global (member) modes of deformation and could be applied to perform either (i) geometrically linear (first-order GBT) or (ii) linear stability (second-order GBT) analyses, thus providing a general and unified approach to obtain accurate and clarifying solutions for a wide range of structural problems. In the last decade, Davies and his collaborators [2] have applied extensively GBT to investigate the buckling behaviour of thin-walled cold-formed steel members and their work provided a strong contribution towards establishing this theory as a valid and often advantageous alternative to fully numerical finite element or finite strip analyses. Indeed, since the GBT d.o.f. are *modal* (instead of *nodal*) displacements, all the member deformed configuration or buckling mode “ingredients” are revealed by the analysis, which still retains the accuracy of the above “conventional” numerical methods.

RECENT GBT FORMULATIONS

In the last few years, the authors developed two major extensions of the GBT formulation originally derived by Schardt, making it possible to perform (i) first-order and linear stability analyses of *arbitrarily orthotropic* thin-walled members, namely laminated plate members made of fibre-reinforced plastic (FRP) layers [3, 4], and (ii) *post-buckling* analyses of cold-formed steel (isotropic) members [5]. Concerning the orthotropic GBT formulation, it was found that, in general, six stiffness matrices (instead of four) are required to describe the cross-section mechanical behaviour. Moreover, due to the orthotropy induced by the laminated plate layer nature and/or configuration, (i) all these matrices (even the "geometric" stiffness matrix) combine both material properties and geometrical characteristics and, unlike in isotropic materials, (ii) both the first and second-order equilibrium equations are strongly coupled. As for the (isotropic) non-linear GBT formulation, it involves significant developments and extensions, with respect to Schardt's "conventional" GBT. Indeed, besides having to deal with the establishment and numerical solution (by means of a standard incremental-iterative procedure) of the non-linear equilibrium equations governing the member large-displacement geometrically non linear behaviour (the conventional GBT equations are only valid for small deformations and moderate rotations), one also needs to account for two additional sets of deformation modes, namely (i) shear and (ii) transverse extension modes (see figure 2). One very interesting feature of the non-linear GBT analyses is the possibility of performing "approximate analyses", including just a selected sub-set of deformation modes, which yield rather accurate results with much less computational effort than the one required to perform "exact" (all-mode) analyses. The traditional finite element or finite strip analyses do not offer such a possibility.

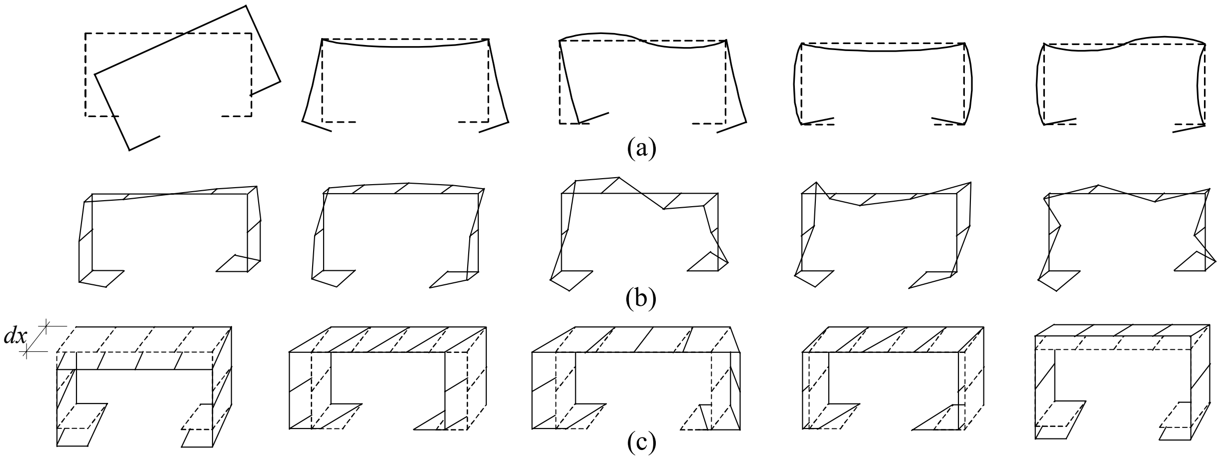

Fig. 2. Sample of (a) conventional, (b) shear and (c) transverse extension GBT deformation modes.

NON-LINEAR ORTHOTROPIC GBT FORMULATION

This paper aims at presenting the derivation and illustrating the application of a non-linear orthotropic GBT formulation, which combines the features of the two formulations briefly described in the previous paragraph, thus making it possible to perform geometrically non linear analyses of laminated plate FRP open-section thin-walled members. Different types of loading and end support conditions can be considered and the theory can handle the presence of initial geometrical imperfections with an arbitrary shape. One is able to determine, with great accuracy, "exact" and "approximate" (*i.e.*, including just a few deformation modes) member post-buckling equilibrium paths, as well as the evolution, along those paths, of (i) the most relevant displacements and stresses and, taking advantage of the GBT unique mode decomposition feature, also of (ii) the deformation mode participation in the member overall deformed configuration. It is worth noting that the deformation mode participation graphs provide a very illuminating insight on the member post-buckling behaviour.

Finally, in order to validate and illustrate the application and capabilities of the formulated non linear orthotropic GBT, several numerical results, concerning the geometrically non linear (post-buckling) behaviour of either pultruded or laminated plate FRP lipped channel columns, beams and beam-columns exhibiting different (orthotropic) material behaviours, are presented and discussed in detail. Some of these results are compared with values obtained from finite element analyses, performed by means of the commercial code ABAQUS and adopting fine shell element meshes to discretise the member.

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