New Trends in Optimal Design of Composite Materials

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Introduction

The current trends in machine design demonstrate evidently that a centre of gravity is continuously moved/shifted from a structural design towards a material design mainly due to an increasing number of new materials (e.g. micro- and nano-composites, functionally graded materials etc.) and a development of new technologies enabling us to produce better and quicker required machines. On the other hand, the material design is mainly directed to the design of a strictly limited group of structures subjected to prescribed boundary and loading conditions and of course such a design should be in advance planned as an optimal one. However, it is worth to emphasise that the optimal material design should take into account not only the required functional properties of designed machines but also the availability and limitations of technological processes and their influence on the final mechanical properties of products.

The aim of the paper is two-fold: (1) to show the possibility of optimisation of technological processes in order to eliminate or to enhance the effects of induced internal stresses/strains during production of multilayered composite structures and (2) using homogenisation approach to periodic structures to discuss the problem of a fibre shape optimisation in order to obtain the required composite material properties. Both problems are analysed in the elastic range only using variational formulations with Lagrange coefficients.

The influence of technological processes

The automated technological processes such as e.g. the resin injection moulding (RIM) or the resin transfer moulding (RTM) introduce internal stresses/strains due to different thermal and chemical shrinkage of fibres and matrices – see Refs [1,2]. The chemical and physical properties of fibres and matrices do not allow to eliminate those effects completely, however, it is possible to reduce them taking into consideration various technological factors – an external pressure is one of them. Using a general formulation for 3-D orthotropic bodies with internal stresses λ_i the stress-strain relations takes the following form:

$$\Omega = \Omega_f \cup \Omega_m \tag{1}$$

The above relation is used to build the functional of total potential energy Π:

$$\Pi = \tfrac{1}{2}\int_{\Omega} \sigma[u(\lambda)]C^{-1}\sigma[u(\lambda)]d\Omega - \int_{\Gamma} pu(\lambda)d\Gamma - \sum_{i=1}^{n}\omega_i A_i\left(\sigma_i^{ad} - \sigma_i - s_i\right) \tag{2}$$

where Ω denotes the space occupied by the body having the boundary Γ, p is an external pressure applied to bond together individual composite plies, ω_i are Lagrange's multipliers, A_i means the area of individual plies and $\left(\sigma_i^{ad} - \sigma_i - s_i\right)$ is a condition of an interlaminar bonding and it has one uknown paramter s_i. The optimisation problem is formulated in the following way:

To find:

$$Min\Pi \tag{3}$$

subjected to constraints:

$$\lambda \le \lambda_0 \tag{4}$$

The constraint condition (4) may be valid for all layers in the laminate or for individual layer.

Using the above formulation the minimisation problem (3) have been solved for multilayered orthotropic plates and cylinders. In mathematical terms the problem is reduced to a system of non-linear algebraic equations. The results demonstrate evidently the possibility (or even necessity) of equalization of internal stresses or possible redistribution of them between layers. Having an information about the required internal stress distribution in the laminate and knowing the functional relation between internal stresses/strains and the parameters of the technological process one can find the optimal set of parameters. The latter problem have been illustrated on the example of the optimal choice of the external pressure distribution.

Shape optimisation of a fibre bundle

In unidirectional composites fibres are not made individual fibres but they constitute in fact a bundle of fibres that is embedded in a matrix material. Using a homogenisation theory for periodic structures a shape of a bundle in each of subcells is assumed/prescribed a priori and it depends on the assumed homogenisation method, i.e. the bundle boundary $\Gamma=\partial\Omega_f$ can be described by a circle, a square, a rectangle or other curve. For our purposes the Suquet homogenisation theory [3] is applied herein. In general, it is based on the introduction of the tensor A. For a given subcell ($\Omega = \Omega_f \cup \Omega_m$)the functional of the total potential energy can be expressed as follows:

$$\Pi = \tfrac{1}{2}\left[C^m_{ijkl} + \left(C^f_{ijkl} - C^m_{ijkl}\right)\left\langle A_{kl\alpha\beta}\right\rangle\right]\varepsilon_{\alpha\beta}\varepsilon_{ij} + \omega\left(\int_{\Omega_f} d\Omega - V_f\right) \tag{5}$$

where the superscripts (or subscripts) f, m corresponds to fibre and matrix, respectively, and V_f means the fibre volume fraction. The optimization problem is formulated in the following way:

For the given displacement field minimize the functional (5) subject to the constraint in the form of the constant fibre volume fraction.

In this way it is possible to control the shape of the bundle of fibres, however, it should be mentioned that the above problem requires additional constraints that refer mainly to the method of composite material production. The bundle of fibres is filled (penetrated) by a matrix material (a resin) and cannot be too thin. Therefore, we propose to introduce the additional condition dealing with the convexity of the boundary Γ. The problem has been formulated in 2-D space. The initial shape of the bundle is assumed in the form of a circle – see Fig.1. The differences in the optimal solutions for two formulations (convex and non-convex boundary) are shown in Fig. 1. The problem has been solved with the use of the FE package NISA II. The bundle boundary is modelled with the help of Bezier's splines. The optimisation method is analogous to that described in Ref. [4]. As it may be seen the additional constraint drastically changes the optimal solutions. However, in order to be closer to practical situations, it is necessary to take also into account the interface layer surrounding fibres. It may have a great influence on a possible failure of composites but its mechanical properties and size are uknown in advance.

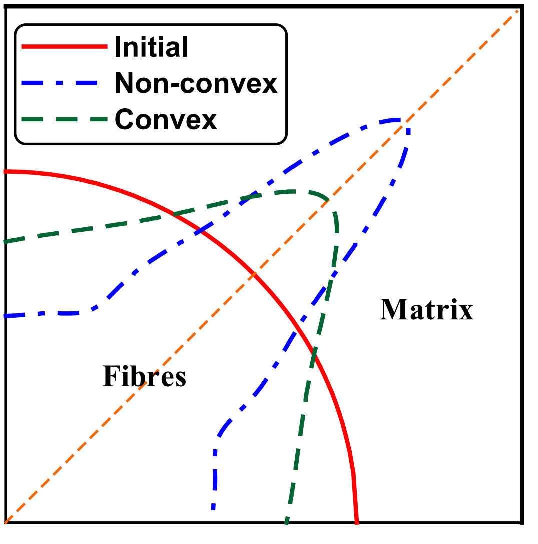

Fig.1 Optimal distributions of fibres in an elementary subcell (a quarter).

References

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