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FRICTIONAL SLIDING OF A MULTISLIP SYSTEM

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Summary The brittle/ductile transition in rocks has an essential influence in determining the strain rate in lithospheric plates. With this motivation we attempt to study the micromechanics of an elastic medium with interfaces whose slip is governed by rate-and-state friction. As a preliminary analysis we study the stability of the steady-state slip of a finite number N of parallel interfaces caused by a constant velocity applied at one edge of the medium. We show that interfacial slip can occur with one or two different slip rates if the steady-state friction law displays a minimum as for dry friction. Our results suggest that, when active slip on all N interfaces is unstable, the medium will select a smaller number of interfaces which continue to slide, the others stopping.

INTRODUCTION

The mechanical behaviour of the earth's lithosphere is broadly described by the combination of a pressure-dependent brittle behaviour and a thermally activated viscoplastic one, called ductile. The brittle behaviour corresponds to the frictional sliding along preexisting faults and can roughly be modelled by a plastic material with a Drucker-Prager criterion. Concerning the mechanics of Plate tectonics, it has been shown that the brittle behaviour is essential because the brittle/ductile transition determines the strain rate of the lithospheric plate in relation to the forces equilibrium driving the plate [2]. This is why we are attempting to understand the micromechanics of such a brittle medium in order to propose homogenized constitutive models. As a preliminary approach, we study the linear stability of the steady sliding of N parallel frictional discontinuities in an elastic body.

THE PROBLEM

We consider a homogeneous elastic medium in mechanical equilibrium of thickness H, infinite in the horizontal direction x and divided by N horizontal interfaces along which friction phenomena occur. We consider rate-and-state friction laws defined by

$$\tau = F(\sigma, \dot{\delta u}, \psi) \quad \text{and} \quad \dot{\psi} = -G(\sigma, \dot{\delta u}, \psi). \tag{1}$$

Such friction laws relate the shear stress τ to the compressive normal stress σ and the rate of slip $\dot{\delta u}$ along the interface. The state of the asperity contacts is modelled by a single state variable ψ. It is supposed that $F_V > 0$, $F_\psi > 0$ and $G_\psi > 0$. The lower boundary is fixed and the upper one moves with a constant horizontal velocity V_H. A constant pressure P is applied on the upper boundary. Assuming the displacement such that $\boldsymbol{u} = u(y,t)\boldsymbol{e}_x$, it is found that the normal stress is homogeneous and equal to $-P$ and that the shear stress depends only on the time: $\tau(t)$. The boundary value problem, the stress continuity on each interface and the Hooke's law give a relation between the displacement $u(y,t)$ in the layer i ($0 \leqslant i \leqslant N$) and all the displacement jumps $\delta u_j(t)$ of each interface $0 \leqslant j \leqslant i$. For $y = H$, the time derivative of this relation leads to a differential equation for the shear stress

$$\dot{\tau} = k\left(V_H - \textstyle\sum_{i=1}^{N} \dot{\delta u_i}\right), \tag{2}$$

where $k = \mu/H$ is a stiffness parameter. Now, regarding the steady-state ($\dot{\tau} = 0$), the previous equation relates the N slip rates by

$$\textstyle\sum_{i=1}^{N} V_i = V_H. \tag{3}$$

We have denoted $V_i = \dot{\delta u_i}$ the steady-state slip rate on each interface. By the equation $(1)_2$ the state of each interface is a function of its corresponding slip rate $\psi_i^{ss} = \psi_i(V_i; P)$. Together with $(1)_1$, it leads to $N-1$ equations $\tau = F^{ss}(V_i; P)$ giving with (3) the nonlinear system to solve to find the steady-state slips V_i. Depending on the properties of the function F^{ss}, different cases can be envisaged. If F^{ss} is a monotonic function of V_i we expect a unique slip rate for the interfaces. If F^{ss} presents an extremum as observed in the case of dry friction [1], two slip rates V_1 and V_2 are expected. If there is a lubrication or a threshold at $V_i = 0$, the interfaces could slip with three different velocities.

MAIN RESULTS OF THE LINEAR STABILITY ANALYSIS

The stability of a steady-state solution is analysed by usual technique consisting of the linearization of the equations of perturbations around the steady state. It yields an eigenvalue problem defined by a matrix depending on a critical stiffness $k_c = F_\psi G_V - F_V G_\psi = -G_\psi \,\mathrm{d}F^{ss}/\mathrm{d}V$, the partial derivatives of F and G and parametrized by k and V_H. It is denoted $\mathsf{M}(k_c^{(l)}, F_V^{(l)}, F_\psi^{(l)}, G_V^{(l)}, G_\psi^{(l)}; k, V_H)$ where l denotes the number of distinct $V_i \neq 0$. Note that $k_c > 0$ corresponds to a velocity-weakening steady-state friction law. It has been shown that the velocity-weakening leads to an unstable sliding with stick-slip in the case of a single interface [4, 3]. We show that $k_c^{(i)}/F_V^{(i)}$ may be an eigenvalue of M whose algebraic multiplicity depends on the number of the interfaces which slip with the same slip rate.

Depending on the stability of the configuration of sliding, *i.e.* one, two or three slip rates, there is a selection of the number of the activated interfaces. We illustrate this result in the case of dry friction in the following.

One steady-state slip rate

When the steady-state friction law is monotonic, only one slip rate for all the interfaces is possible. Then $V_i = V_H/N\ \forall i$ because of (3). Hence, the determinantal equation is $(k_c/F_V - s)^{N-1}\left[s^2 - s\,(k_c - Nk)/F_V + NkG_\psi/F_V\right] = 0$. If $N > 1$, we show that if $k_c < 0$ the sliding is stable for all the interfaces. If $k_c > 0$, the biggest eigenvalue is k_c/F_V and the slip is unstable. It is important to note that, in this configuration, the most unstable mode is real and then the stick-slip instability as in the single-interface problem is not the most unstable mode. Looking at the associated eigenvector, it is remarkable that the presence of many interfaces allows the system to be unstable while the shear stress remains constant. The instability criterion of this configuration is then $k_c > 0$. That is, the slip rate V_H/N has to correspond to a velocity weakening behaviour of the friction law. About the evolution of an unstable system, the eigenvector associated to k_c/F_V suggests that $N-1$ of the interfaces have to slow down whereas one of them has to speed up. Using the Dieterich-Ruina law, this is confirmed by a direct numerical integration of the dynamical system (1)–(2) starting from an initial condition near the steady state. We then expect that an unstable system evolves from a configuration with N interfaces sliding at V_H/N towards a system which slips at V_H just on one interface. The slip is localised. Then, depending on the value of k this single-interface sliding can be unstable and present a Hopf bifurcation. A question then arises: because the shear stress has to increase, is this stick-slip instability able to promote a slip on another interface? It is impossible in the case of friction laws like the Dieterich-Ruina one because of its logarithmic singularity and the unbounded evolution of ψ.

Two steady-state slip rates

We consider a convex steady-state friction law with a minimum at $V = V_m$ so that two steady-state slip rates can exist: one in the creep regime, the other in the inertial one (cf. Fig. 1). By convention $V_1 < V_2$. Then, n interfaces slip at V_1 and m at V_2 such that $n+m = N$ and $nV_1 + mV_2 = V_H$ by (3). This relation, together with the steady-state condition $F^{ss}(V_1) = F^{ss}(V_2)$, has the obvious solution $V_1 = V_2 = V_H/N$ which is stable only if $V_H > NV_m$. A solution $V_1 \neq V_2$ can only exist if the curve $\tau^{ss}(V^{ss})$ is asymmetric around V_m. If the strengthening part of $\tau^{ss}(V^{ss})$ is steeper (resp. less steep) than the weakening part, a supercritical (resp. subcritical) pitchfork bifurcation is expected to occur at $V_H = NV_m$ (Fig. 2).

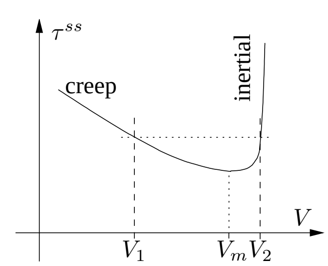

Figure 1. Dry friction law (cf. [1])

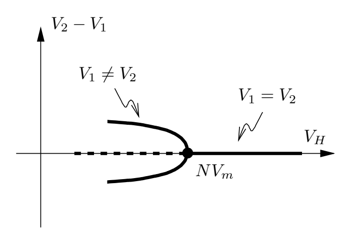

Figure 2. Bifurcation diagram of the two slip-rate solution

In fact, the linear stability analysis shows that the solution $V_1 \neq V_2$ is unstable when there is more than 1 slow interface ($n > 1$) and stable otherwise. Indeed, the roots of M are given by the polynomial $P_3(s)\left(s_1 - s\right)^{n-1}\left(s_2 - s\right)^{m-1} = 0$ where $P_3(s)$ is a cubic and $s_i = k_c^{(i)}/F_V^{(i)}$. Thus, 5 eigenvalues are expected. Using the Routh-Hurwitz criterion to study the location of the P_3's roots, we show that the mode s_1 is the most unstable for $k_c^{(1)} > 0$. Provided $n \neq 1$, this mode leaves the shear stress τ constant and suggests that $n-1$ interfaces stop and one of them speeds up. The total number of slipping interfaces changes. We then guess that such a multislip system should evolve towards a system with only one slow interface ($n = 1$) because s_1 is no longer a mode. It can be further shown that the solution $V_1 \neq V_2$ ($V_H < NV_m$) then is stable for any stiffness k, at least when V_H is close to NV_m.

CONCLUSIONS

We show that the sliding of a heterogeneous medium holding a large number of dry frictional interfaces depends strongly on the nature of the steady-state rate-and-state friction law. A purely velocity-weakening law implies that the multislip is unstable and only one slipping interface is selected. On the other hand, if the steady-state law presents a minimum, there is a critical driving velocity $V_H^c = NV_m$ (*i.e.* a critical number of interfaces $N_c = \lfloor V_H/V_m \rfloor$) above which (*i.e.* such that $N \leq N_c$) the multislip system is stable, all interfaces sliding at V_H/N. Below V_H^c, a stable multislip system with 1 slow interface and $N-1$ faster ones could exist if the velocity-strengthening part of the friction law is steeper than the weakening part. Thus, for this multislip system, the nature of the instabilities is different from the stick-slip oscillations involved in the single interface problem and it could lead to interesting experimental investigations in order to constrain the friction law around its minimum and get a better understanding of what happens when $V \to 0$. Furthermore, the richness of behaviours of this system should give new insights concerning earthquake dynamics and the localisation of deformation in brittle media.

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